APPLICATION OF HIGHER ORDER SPECTRAL METHOD FOR DETERMINISTIC WAVE FORECAST

Günter F. Clauss  
Ocean Engineering Division  
Technical University Berlin  
Germany  
clauss@naoe.tu-berlin.de

Marco Klein  
Ocean Engineering Division  
Technical University Berlin  
Germany  
klein@naoe.tu-berlin.de

Matthias Dudek  
Ocean Engineering Division  
Technical University Berlin  
Germany  
dudek@naoe.tu-berlin.de

Miguel Onorato  
Dipartimento di Fisica  
Università di Torino  
Italy  
miguel.onorato@unito.it

ABSTRACT

This paper addresses the Higher Order Spectral (HOS) method as very fast and accurate non-linear method for deterministic wave forecast. The focus of the paper lies on wave propagation, with the objective to draw conclusions on the applicability of the HOS method for deterministic wave forecast. Systematic numerical and experimental investigations are conducted. The investigations comprise exact solutions of the non-linear Schrödinger equation (NLS) as special non-linear wave groups, which are used as initial conditions for the subsequent simulations. Different parameters such as relative water depth, wave steepness and propagation distance are varied to evaluate the applicability of the HOS method. The results obtained are validated by experiments as well as fully non-linear simulations. It is shown that the HOS method is capable for non-linear wave forecast due to high accuracy and fast calculation time at once.

INTRODUCTION

The scope of application of offshore structures depends on limiting criteria such as absolute or relative motions - from efficient and economic offshore operations in moderate sea states to reliability as well as survival in extreme wave conditions. These predefined criteria are typically combined with a limiting characteristic wave height or sea state based on stochastic analysis in the design process. The stochastic analysis yields the probability of exceeding a specific threshold for certain sea states. But most sea states (in particular in the transition area between feasible and unfeasible region of a scatter diagram) will feature favourable wave sequences allowing short-term offshore operations which are elapsed unused. Thus, a decision support system based on deterministic wave and motion prediction for short-term offshore operations can reduce the downtime. Particularly with regard to the challenges of offshore wind energy, with numerous operations during installation and maintenance, a deterministic decision support system will lead to a significant increase of the efficiency of the involved offshore structures.

One principal point of the development process of such a system is the implementation of very fast algorithms to obtain a sufficient wave and motion prediction time span in advance. Thereby, three main constituents have to be assembled - sea state registration, wave forecast and motion prediction. This paper focuses on the middle part - the wave forecast based on surface elevation snapshots taken continuously by a ship board radar at

Copyright © 2014 by ASME
great distance ahead the operational area. These snapshots, pre-
processed by the wave monitoring system WaMoS II® (Wave
and Surface Current Monitoring System), can be used as input
for the wave forecast tool.

Due to contrary specifications on the wave forecast tool -
very fast calculation time and high accuracy at once - only few
methods are capable. The linear wave theory, which is the fastest
method but also simplest one, has already been utilized for wave
forecast applications with promising findings [1–8]. German-
ischer Lloyd developed a Shipboard Routing Assistance system
(SRA) that is based on the continuous shipboard measurement of
the surrounding seaway, using the ships X-band radar [1]. Claus-
et al. [2, 4, 6] as well as Kosleck [8] implemented a decision sup-
port system for Computer Aided Ship Handling (CASH) based
on linear evolution of surface elevation snapshots measured con-
tinuous using also the ships X-band radar. The tool has been
developed for long crested waves at beginning, showing that the
encountering wave field as well as the structure response can be
predicted fairly accurate in moderate sea states [2]. Later, the tool
is enhanced to short crested wave prediction calculating the en-
countering wave field sector-wise resulting in the DOF motions
in advance [4, 6, 8]. Thereby, a new adaptive pressure distribu-
tion method has been introduced for calculating the pressure distribu-
tion on the ship hull as a result of the current sea state. This
allows the determination of the global and local loads in time
which can be used to detect critical local or global load cases [8].
Also Naaijen and Huijmsmans [3, 7] as well as Naaijen et al. [5]
investigated real time wave forecasting for ship motion estima-
tion based on linear evolution equations (for long as well as short
crested waves). They concluded, that a 60 s accurate forecast of
wave elevation is very well feasible for all considered wave con-
ditions. Motion predictions are even more accurate.

But the linear approach implies uncertainties due to its
strong simplifications of the water wave problem. The inaccuracy
increases with increasing wave steepness as non-linear ef-
teffects become dominant. Higher order non-linear methods allow
sophisticated simulations at advanced level but at the expense of
computation time. Wu [9] as well as Blondel et al. [10] pre-
sented the deterministic reconstruction and prediction of non-
linear wave fields using the HOS method. The basic idea is to use
the HOS method for the wave evolution in time. But the start-
ing point of their studies is the reconstruction of the wave field in
space domain (required as initial condition of the HOS method)
based on one or several wave probes at specific locations. So,
sophisticated optimization procedures have to be used to obtain
clusion on the wave field in space based on registrations in
time. This reconstruction process showed to be the drawback
for an effective application of the HOS method for wave fore-
casting as this process is very time-consuming. But it has been
shown that the HOS method in principle is an effective approach
for long-time and large-space simulation of non-linear wave-field
evolutions due to its high efficiency and accuracy [9].

This paper presents the application of the HOS method for
deterministic wave forecast by taking advantage of the supposed
drawback. Surface elevation snapshots taken continuously by a
ship board radar at great distance will be directly used as input.
Thus, the efficient and accurate method can be directly util-
ized without time-consuming reconstructions of the wave field
in space domain. The focus of the paper lies on the implementa-
tion and validation of the HOS method for arbitrary water depth.
For this purpose special non-linear wave groups, envelope soli-
ton and Peregrine breather solutions of the NLS equation, are
investigated systematically. Thereby, parameters such as wave
steepness, relative water depth as well as propagation distance
are varied. The results obtained are validated by experiments as
well as fully non-linear simulations.

HIGHER-ORDER SPECTRAL METHOD

The HOS method has been introduced independently by
West et al. [11] and Dommermuth and Yue [12] (cf. Tanaka [13]).
This procedure takes all non-linear interactions, resonant and
non-resonant, into account. For our investigation the numerical
procedure presented in West et al. [11] has been implemented,
which is briefly described in the following. The fluid is assumed
to be incompressible, inviscid and irrotational. In addition, long-
crested waves are considered exclusively. A velocity potential φ
can then be introduced as \( \bar{u} = \nabla \phi \), where \( \bar{u} \) is the fluid velocity
field. Using the incompressibility condition, it is easy to show
that the velocity potential satisfies the Laplace equation,

\[ \nabla^2 \phi = 0. \]  

The dynamic and kinematic boundary conditions on the free sur-
face \( z = \zeta(x,t) \) take the following form, respectively:

\[ \frac{\partial \phi}{\partial t} + \frac{1}{2} ( \nabla \phi )^2 + g \zeta = 0, \]  

\[ \frac{\partial \zeta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \zeta}{\partial x} = \frac{\partial \phi}{\partial z}. \]

Assuming that the bottom, \( z = -h \), is flat, then the boundary con-
dition states that the vertical velocity is zero on the bottom.

The HOS method is based on the series expansion of the poten-
tial velocity around the surface. At the beginning, the poten-
tial equations are converted to equations at the free surface
\( \Psi(x,t) = \phi(x, \zeta(x,t), t) \), resulting in the following kinematic,

\[ \frac{\partial \zeta}{\partial t} = - \nabla_x \Psi \cdot \zeta_x + W(1 + (\nabla_x \zeta)^2), \]  

Copyright © 2014 by ASME
and dynamic boundary condition,

\[
\frac{\partial \Psi}{\partial t} = -g\zeta - \frac{1}{2}(\nabla_x \Psi)^2 + \frac{1}{2}W^2(1 + (\nabla_x \zeta)^2),
\]

(5)

where \(W = \partial \Phi|_{z=\zeta}\) is the vertical velocity at the free surface and \(\nabla_x\) is the gradient operator in \(x\)-direction. The main problem of solving Eq. 4 and Eq. 5 is to find a solution for \(W\) in terms of \(\zeta\) and \(\Psi\). The procedure proposed by West et al. [11] starts from the formal expression that the velocity potential \(\Psi(x,t)\) can be represented as Taylor series expansion at \(z = 0\),

\[
\Psi(x,t) = \sum_n \frac{\xi^n}{n!} \frac{\partial^n}{\partial z^n} \Phi(x,0,t).
\]

(6)

Accordingly, the vertical velocity reads

\[
W(x,t) = \sum_n \frac{\xi^n}{n!} \frac{\partial^{n+1}}{\partial z^{n+1}} \Phi(x,0,t).
\]

(7)

Now, the problem is transformed to the reference function \(\Phi(x,0,t)\) to be solved for the boundary value problem at \(z = 0\). Knowing \(\Psi(x,t)\), the reference function can be obtained by formally inverting Eq. 6. Afterwards, the vertical velocity at the free surface is determined from the infinite series of Eq. 7. Thereby, formal expansions for \(\Psi\),

\[
\Psi(x,t) = \sum_{n=0}^{m} \phi^{(n)}(x,z,t),
\]

(8)

and \(W\),

\[
W(x,t) = \sum_{n=0}^{m} W^{(n)}(x,z,t),
\]

(9)

are introduced to solve Eqs. 6 & 7. \(\phi^{(n)}\) is assumed to be of order \(O(\zeta^n)\), with \(\zeta\) as ordering parameter and \(M = m + 1\) being the order of approximation of non-linearity. Insertion of Eq. 8 in the inverted Eq. 6 and separating the terms of each order (assuming that the problem can be solved independently for every order \(n\)) yields

\[
\phi^{(0)}(x,0,t) = \Psi(x,t),
\]

(10)

for the first order. The next higher order solutions are obtained successively from the next lower order solution,

\[
\phi^{(n)}(x,0,t) = -\sum_{k=1}^{n} \frac{\xi^k}{k!} \frac{\partial^k}{\partial z^k} \phi^{(n-k)}(x,0,t).
\]

(11)

From Eq. 7 the vertical velocity \(W\) at the surface \(\zeta\) is obtained

\[
W^{(n)}(x,t) = -\sum_{k=0}^{n} \frac{\xi^k}{k!} \frac{\partial^{k+1}}{\partial z^{k+1}} \Phi^{(n-k)}(x,0,t).
\]

(12)

In arbitrary water depth \(\phi^{(n)}(x,z,t)\) can be derived by

\[
\phi^{(n)}(x,z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi^{(m)}(k,t) \cosh(|k|(z+h)) e^{ikx} dk.
\]

(13)

If periodic boundary conditions are assumed, the Fast Fourier Transform (FFT) algorithm can be utilized for determining \(\phi^{(n)}(x,z,t)\), reducing the number of operations significantly. This has been applied for this study, as a fast calculation time is an important prerequisite for the wave forecast tool. To avoid Fourier space aliasing for the higher order terms, a low pass filter according to West et al. [11] has been implemented. In this study, the series expansion has been expanded up to the fourth order (\(M = 4\)). In addition, an exponential damping term has been introduced in the Fourier space to suppress high frequency contamination occurring for the highest waves, which can cause numerical instabilities. The damping term has been implemented in such a way, that the Fourier coefficients at a specific wave number \(k_{damp}\) are damped by an exponential term which reaches zero at the low pass filter limit. The specific wave number has been set to \(k_{damp} = 40 \text{rad/m}\) for this study after systematic investigations on different wave steepness. This value showed no influence on lower wave steepness, but stabilized the simulations with the highest wave steepness, which aborted without. But the procedure is neither capable to handle wave breaking effects nor to simulate such steep waves close to breaking.

The principal procedure is as follows: Eqs. 11 and 12 are determined, \(W^{(n)}\) is then inserted in the time evolution equations (Eqs. 4 and 5) and the equations are advanced in time. The substitution of \(W^{(n)}\) into the equations has to be done carefully. The consistency of ordering has to be retained and is counted as the power of \(\zeta\) or its gradient, occurring in the terms associated with \(W^{(n)}\) [11]. In this study, the time evolution equations are advanced with the fourth order Runge-Kutta-Gill method.

**VALIDATION**

The objective of the validation of the above presented numerical procedure is to draw conclusion on the applicability for deterministic wave forecast. A typical application of such a tool implies the numerical evolution of a surface elevation snapshot (taken from ship board radar) over large distances up to the off-shore structure. Hence, the focus of the investigation lies on the
accuracy of the prediction of wave group propagation over large distances. Thereby, the influence of the level of non-linearity of the wave group as well as the influence of the water depth on the accuracy of the wave propagation prediction is investigated.

Two different exact solutions of the NLS equation – the envelope soliton and the Peregrine breather solution – are utilized to investigate the capability of the HOS method to simulate steep non-linear wave groups accurately. Both solutions are predestined for evaluating the non-linear wave-wave interactions.

Soliton wave groups remain stable over large distances, due to balancing the dispersive against the focusing behaviour of the wave group. The soliton solution is used to investigate the influence of the wave steepness on the accuracy of the numerical program, i.e. systematic variation of the initial steepness of the soliton up to an extreme value of $k_0 \cdot A_0 = 0.3$.

In contrast to that, breather solutions are unstable and evolve to a wave group with a steep, high single wave. The Peregrine breather is used to validate the numerical procedure against the water depth influence on the non-linear wave-wave interactions, i.e. systematic variation of the water depth using the same Peregrine breather solution.

**Semi-Experimental Procedure**

The HOS method requires a surface elevation snapshot and evolves the waves in space domain. This is advantageous for the intended forecast tool as the radar snapshots can be directly taken for the subsequent simulations, but complicates the validation with experimental data from a seakeeping basin. Time-consuming and expensive series of measurements would be necessary to detect the surface elevation in space domain in the seakeeping basin. The reason for this is that plenty of successive, pointwise measurements have to be conducted (with suitable calm down time in between) to obtain the snapshot.

To avoid this issue, a semi-experimental approach has been applied, enabling significant fewer measurements by more validation scenarios at the same time. The required surface elevation snapshots for the HOSM simulation are determined using the fully non-linear numerical wave tank WAVETUB [14], as the seakeeping basin at Technische Universität Berlin (TUB) can be modelled exactly with this numerical wave tank (including the wave maker). Due to the same geometry of the wave maker, the boundary condition of the wave board motion is identical and ensures that the starting point of the wave evolution is also identical for both tanks. Selected validation scenarios are investigated in the seakeeping basin, enabling the pointwise validation of the input snapshot as well as of the HOSM simulation results by measurements in time domain at fixed positions (near the wave maker to validate the input wave sequence and far away to validate the HOSM result). In addition, the HOSM simulation results can also be compared to WAVETUB results exclusively, which allows the variation of the (numerical) wave tank configurations, i.e. the keywords are water depth and wave tank lengths, which can not be varied in the physical wave tank.

Following, a brief description of the seakeeping basin as well as the numerical wave tank is given:

**Seakeeping Basin** The experiments are performed in the seakeeping basin of the Ocean Engineering Division at TUB. The basin is 110 m long, with a measuring range of 90 m. The width is 8 m and the water depth is 1 m. On the one side, an electrically driven wave generator is installed, which can be used in piston as well as flap-type mode. The wave generator is fully computer controlled and a software is implemented which enables the generation of regular waves, transient wave packages, irregular sea states as well as tailored (critical) wave sequences. On the opposite side, a wave damping slope is installed to suppress disturbing wave reflections.

**WAVETUB** This potential theory solver has been developed at TUB for the simulation of non-linear wave propagation [14, 15]. The two-dimensional, non-linear free surface flow problem is solved in time domain: the fluid is considered inviscid, incompressible, and the flow is irrotational. The atmospheric pressure above the free surface is constant and surface tension is neglected. Hence, the flow field can be described by a velocity potential which satisfies the Laplace equation for Neumann and Dirichlet boundary conditions. At each time step, a new boundary-fitted mesh is created and the velocity potential is calculated in the entire fluid domain, using the Finite Element method. On the basis of this solution, the velocities at the free surface are determined by second-order differences. For long term simulations, a numerical beach is implemented at the end of the wave tank by adding artificial damping terms to the kinematic and dynamic free surface boundary condition. To develop the solution in time domain the fourth-order Runge-Kutta formula is applied. For the generation of the waves, a moving wall is implemented at one side of the numerical wave tank, which enables the simulation of piston-type, flap-type and double flap-type wave boards. A detailed description of the theoretical background can be found in [14]. WAVETUB is an established program which shows a high accuracy over the past years for a multitude of different tasks (cf. [14–19]).

**Envelope Soliton**

The first part of the validation program comprises the investigation of envelope solitons. Envelope solitons are exact solutions of the NLS equation and they can be found for example using the Inverse Scattering Transform developed by Zakharov and Shabat [20] theoretically. Yuen and Lake [21] verified their existence experimentally. They are widely used as basic model which describes weakly non-linear wave water propagation in one direction and are utilized for basic experimental [21–23] as well
as numerical investigations [11, 12, 24]. The general conclusion of these studies is that they propagate with permanent form constituting a stable wave group by balancing the wave dispersion and non-linearity of the system, if transverse direction effects are neglected. The investigations indicated also that relatively steep envelope solitons \((k_0 \cdot A_0 \geq 0.2)\) will show dispersive behaviour. But recently, fully non-linear simulations by Dyachenko and Zakharov [25] exposed very steep, stable wave groups (up to the wave breaking limit), containing only a small number of individual steep waves. Slunyaev et al. [26] verified their existence in laboratory experiments, showing very steep envelope solitons \((k_0 \cdot A_0 \approx 0.3)\) remaining stable in the wave tank and exhibit neither noticeable radiation nor structural transformation for more than 60 wavelengths. In addition, it has been shown that the envelope soliton solution of the NLS equation is a reasonable first approximation for specifying the wave-maker driving signal in laboratory tests.

Moreover, the solitons are an excellent first benchmark test for the validation of the HOS method. In the following, three different soliton solutions – from moderate wave steepness \((k_0 \cdot A_0 = 0.15)\) to a very steep wave group \((k_0 \cdot A_0 = 0.30)\) – are investigated. The three solitons are chosen from the experiments performed in Slunyaev et al. [26]. They share the same features in terms of wave length and relative water depth, but cover a large range of wave steepness – cf. Tab. 1. In addition, they are characterized by a very stable behaviour over a large distance in the wave tank. During the experiments ten wave gauges are used - from 10 m to 85 m in front of the wave maker. For our investigations only two wave gauges are of interest. The first one, at \(x = 10 m\), is used to validate the WAVETUB simulation close to the wave maker and the last one, at 85 m, is used to validate the HOSM results.

Figure 1 presents the experimental and numerical development of the three different soliton solutions. The diagram is arranged as follows: the three different wave steepness are illustrated separately (black rectangles – top for test number 29.14 \((k_0 \cdot A_0 = 0.15)\), centre for test number 30.13 \((k_0 \cdot A_0 = 0.23)\) and bottom for test number 30.37 \((k_0 \cdot A_0 = 0.30)\)). The top diagram of each block presents the HOSM input snapshot calculated with WAVETUB, the centre diagram of each block compares the experimental data with the WAVETUB results at the beginning of the evolution \((x = 10 m)\) and the bottom diagram compares the experimental data with the HOSM result at the end of the evolution \((x = 85 m)\). Note that the ordinate axis are equally scaled for a better comparison.

The increasing of the wave steepness from top to bottom is evident, whereby the wave group height increases and the number of waves within the wave group decreases. The comparison between the measurements and the WAVETUB calculation at the beginning of the evolution \((x = 10 m)\) show that the formation process of the wave group is reproduced accurately. Hence, WAVETUB can be employed for the determination of the input snapshot for the subsequent HOS simulation. Slight deviation can only be observed for the steepest case, the highest wave is not fully captured by the numerical wave tank.

The bottom diagram of each block shows that the HOS method yields stable envelope solitons with a very similar shape as measured in the wave tank at \(x = 85 m\). Thereby, it is remarkable that the simulation distance is approximately fifty wave length. Again, the steepest case shows some deviations in amplitudes of the highest wave. Now, the HOSM simulation yields higher values, showing that the physical wave seems to lose some energy during propagation along the tank.

Altogether, the HOS method predicts the evolution of the non-linear wave group over large distances very accurately – both shape of the wave group and wave height. Also the time of occurrence of the wave group deviates only slightly \((dt \leq 0.3 s)\).

**Peregrine Breather**

The second part of the validation program comprises investigations of Peregrine breathers. The Peregrine breather is one of three know exact breather solutions of the NLS equation. The Peregrine solution, also known as rational solution, has been originally proposed by Peregrine [27]. It has the peculiarity of being not periodic in time and in space: it is a wave that appears from nowhere and disappears without trace [28]. The breather is characterized by a small perturbation of a plane wave (Stokes wave) in the one dimensional water wave problem. Due to modulation instability, also known as the Benjamin-Feir instability [29, 30], the breathers grows exponential, reaching a maximum amplitude and vanishes back to the plane wave solution. The maximum amplitude the breather reaches during propagation is three times the amplitude of the unperturbed waves (plane wave solution). The Peregrine solution has been recently reproduced experimentally in wave tank laboratories [31, 32].

The previous paragraph showed that the HOS method is capable to predict non-linear wave evolution over large distances, showing that also very steep wave groups can be simulated. Now, the Peregrine breather is unstable due to modulation instability and evolves to a wave group with a steep, high single wave three times the surrounding waves. Previous investigations [19] showed that the water depth influences the breather evolution. The experiments as well as the numerical simulations reveal that

**TABLE 1.** Overview on the investigated envelope soliton solutions.

<table>
<thead>
<tr>
<th>test number</th>
<th>(k_0 \cdot A_0)</th>
<th>(\omega_0 \text{ (rad/s)})</th>
<th>(k_0 \cdot d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.14</td>
<td>0.15</td>
<td>6.86</td>
<td>4.8</td>
</tr>
<tr>
<td>30.13</td>
<td>0.23</td>
<td>6.86</td>
<td>4.8</td>
</tr>
<tr>
<td>30.37</td>
<td>0.3</td>
<td>6.82</td>
<td>4.74</td>
</tr>
</tbody>
</table>
FIGURE 1. DEVELOPMENT OF THE THREE DIFFERENT SOLITON SOLUTIONS ALONG THE WAVE TANK.
limited water depth influences the formation process significantly. This fact is used to validate the HOS method against limited water depth effects, as the extreme wave evolution will show significantly deviations for decreasing water depths close to \( k_0 \cdot d \to 1.36 \). Thus, this should also be predicted by the HOS method, when the water depth influence is modelled correctly.

As aforementioned, the input snapshots for the HOSM simulations are again obtained by numerical calculations using WAVETUB. The WAVETUB and HOSM results are not compared with experimental data, as the varying water depth as well as the required distance up to the point of maximum amplitude can not be covered by the test facility. Therefore numerical calculations with WAVETUB for different water depths using the same Peregrine breather are performed exclusively. Table 2 presents the investigated breather solutions. The surface elevation is kept constant at the wave board and the water depth is varied from \( d = 1 \, \text{m} \) via \( d = 2 \, \text{m} \) to \( d = 4 \, \text{m} \). The wave board motion has been modified for each water depth, using the Biesel function [33] to generate identical wave sequences. The Biesel function relates the wave board stroke to the wave amplitude at the position of the wave maker in dependency of the water depth and geometry of the wave maker.

To assess the accuracy of the numerically reproduced wave sequences for the different water depths – in particular as the initial steepness plays a major role for the formation of the highest wave – registrations at \( x = 10 \, \text{m} \) in front of the wave maker for each realization are compared in frequency domain. The distance of \( x = 10 \, \text{m} \) is chosen to ensure that the generated wave is completely developed on one hand and that the location is close to the wave maker on the other hand, so the spectrum of the wave sequence does not change significantly due to the modulation instability. Figure 2 presents the comparison of the spectra for the different water depths. It is obvious that the agreement between the different wave spectra for the different water depths is excellent. From this comparison, it can be deduced that the wave sequences generated for the different investigated water depths are almost identical at the wave board.

Figure 3 presents the development of the Peregrine breather in the three different water depths. The diagram is arranged as follows: the consequences of the three different water depths on the breather propagation are illustrated separately (black rectangles – top for water depth \( d = 4 \, \text{m} \), centre for water depth \( d = 2 \, \text{m} \) and bottom for water depth \( d = 1 \, \text{m} \)). The top diagram of each block presents the HOSM input snapshot calculated with WAVETUB and the bottom diagram of each block compares the HOSM snapshot with the WAVETUB snapshot at the time of maximum amplification of the breather.

The bottom diagram of each block demonstrates that the distance, the breather has to pass before reaching its maximum wave height, is strongly depending on the water depth, which is in accordance with previous investigations [19]. Thereby the HOS method provides very similar results compared to the fully non-linear simulations, showing that the water depth influence on wave propagation is modelled accurately. In intermediate water depth (\( d = 1 \, \text{m} \)), the breather travels more than twice the distance compared to deep water to reach its maximum crest height. This shows that the HOS method can be used for the simulation of wave evolution over large distances, as the simulation distance amounts between 34 carrier wave lengths for \( d = 4 \, \text{m} \) and 113 carrier wave lengths for \( d = 1 \, \text{m} \). The results indicate that even longer distances can be practicable. In addition, the results show that steep single waves can be simulated and thus detected by the tool. The zero-upcrossing steepness of the maximum wave reaches \( \pi \cdot H/L \approx 0.22 \) for the three different water depths, which is approximately three times the initial steepness and thus also the theoretical maximum of the used Peregrine input solution.

To compare the evolution of the breather in general, in particular the evolution process of the highest wave, the concept of maximum temporal amplitude (MTA) is applied for both methods [34]. The time-independent MTA is defined as maximum

**TABLE 2.** Overview on the investigated Peregrine breather solutions.

<table>
<thead>
<tr>
<th>water depth [m]</th>
<th>( k_0 \cdot a_0 )</th>
<th>( \omega_0 ) (rad/s)</th>
<th>( k_0 \cdot d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.075</td>
<td>4.48</td>
<td>2.05</td>
</tr>
<tr>
<td>2</td>
<td>0.075</td>
<td>4.48</td>
<td>4.1</td>
</tr>
<tr>
<td>4</td>
<td>0.075</td>
<td>4.48</td>
<td>8.2</td>
</tr>
</tbody>
</table>
FIGURE 3. DEVELOPMENT OF THE PEREGRINE BREATHER IN THE THREE DIFFERENT WATER DEPTHS – THE WATER DEPTH INCREASES FROM BOTTOM TO TOP. THE RESULTS OF THE THREE DIFFERENT WATER DEPTHS ARE ILLUSTRATED SEPARATELY MARKED WITH BLACK RECTANGLES – TOP FOR WATER DEPTH \( d = 4 \) m, CENTER FOR WATER DEPTH \( d = 2 \) m AND BOTTOM FOR WATER DEPTH \( d = 1 \) m. THE TOP DIAGRAM OF EACH BLOCK PRESENTS THE INPUT SNAPSHOT CALCULATED WITH WAVE TUB AND THE BOTTOM DIAGRAM COMPARES THE HOSM RESULT WITH THE WAVE TUB RESULT IN SPACE DOMAIN AT THE TIME OF MAXIMUM AMPLIFICATION OF THE BREATHER.
surface elevation in space of a travelling wave sequence over time, i.e. the maximum surface elevation at each point in space is determined over the simulation time. Figure 4 presents the development of the MTA for the three different water depths – the water depth increases from bottom to top. It is obviously that the HOS method also provides similar results for the growth of the highest wave in space compared to the fully non-linear simulation. It can be deduced from the diagrams that the water depth influences both, the growth rate as well as the formation of the breather itself at the beginning of the evolution. The wave group with the highest wave settles down on a lower wave height level for decreasing water depth at the beginning of the evolution. The reason for this is that the Peregrine solution is identical at the wave board for the three water depths (same wave sequences via adapted wave board motions - cf. Fig. 2), which results in an adjustment of the shape of the breather at the beginning of the evolution according to the real water depth. But the water depth influence on the growth rate is the main reason for the different evolution of the breathers. Both effects are captured by the HOS method and somewhat more significant deviations between HOSM and WAVETUB can only be observed for very long simulation distances in the bottom diagram.

**CONCLUSION**

This paper presents a numerical and experimental study on the HOS method for the application as deterministic wave forecast tool. The focus lies on the predictability of non-linear wave group evolution. For the investigations, two exact solutions of the NLS equation are utilized – the envelope soliton solution and the Peregrine breather solution. The soliton solution has been applied to evaluate the HOS method regarding the impact of the wave steepness on the wave evolution. The Peregrine breather solution is investigated in different water depths to take the water depth influences on the non-linear wave propagation in the HOSM simulation into account. The results are validated by a semi-experimental procedure – wave tank experiments as well as fully non-linear simulations are considered.

It is shown that the HOS method predicts the evolution of non-linear wave groups over large distances very accurately – both shape of the wave group and wave height. The method is capable to simulate the evolution of very steep waves – even an envelope soliton with a wave steepness of $k_0 \cdot A_0 = 0.30$ has been predicted with sufficient accuracy compared to wave tank measurements. Thereby, it is remarkable that the simulation distance of this steep soliton is approximately fifty wave lengths. In addition, it has been shown that the water depth influence on the formation process is also modelled accurately in the numerical procedure. The HOS method predicts the non-linear dynamics of a Peregrine breather in different water depths very similar compared to the fully non-linear approach, showing that steep single waves can be simulated and thus detected by this method. Again, the simulation comprises the wave evolution over large distances as the simulation distance amounts between 34 carrier wave lengths for $d = 4\, \text{m}$ and 113 carrier wave lengths for $d = 1\, \text{m}$. The results also indicate that even longer distances can be practicable.

However, the major outcome of this investigation is that the tool predicts the non-linear wave evolution very accurate. Thereby, steep wave events as well as intermediate water depth are modelled accurate enough for applying the intended wave forecast tool for a wide application range. Thereby, the computational time required for the HOSM simulation throughout the study showed great promise for being fast enough for a full scale wave forecast application.

The next steps are systematic investigations of irregular sea states as well as full scale validations. Irregular sea states have to be investigated in terms of sea state parameters (significant wave height $H_s$, peak period $T_p$, spectral bandwidth $\gamma$, etc.). The water depth influence is thereby again one key aspect as the future application will lie in offshore operations at offshore wind parks which are basically located in intermediate water depth.

**ACKNOWLEDGMENT**

This paper is published as a contribution to the joint research project "PrOWOO". The authors wish to express their gratitude to the German Federal Ministry of Economics and Energy (BMWi) and Project Management Jülich (PtJ) for funding and supporting the joint research project. We highly acknowledge the support of this research project and want to thank our project partner OceanWaveS.

M. Onorato was also supported by ONR Grant No. 214

Copyright © 2014 by ASME
REFERENCES


[26] Slunyaev, A., Clauss, G. F., Klein, M., and Onorato, M.,


