ABSTRACT

The existence of freak waves is indisputable due to observations, registrations, and severe accidents. The occurrence of extreme waves, their characteristics and their impact on offshore structures is one of the main topics of the ocean engineering research community. Real sea measurements play a major role for the complete understanding of this phenomenon. In the majority of cases only single point registrations of real sea measurements are available which hinders to draw conclusions on the formation process and spatial development in front of and behind the respective registration points.

One famous freak wave is the "New Year Wave", recorded in the North Sea at the Draupner jacket platform on January 1st, 1995. This wave has been reproduced in a large wave tank and measured at different locations, in a range from 2163 m (full scale) ahead of to 1470 m behind the target position – 520 registrations altogether. Former investigations of the test results reveal freak waves occurring at three different positions in the wave tank and these extreme waves are developing mainly from a wave group. The possible physical mechanisms of the sudden occurrence of exceptionally high waves have already been identified - superposition of (nonlinear) component waves and/or modulation instability (wave-current interaction can be excluded in the wave tank).

This paper presents experimental and numerical investigations on the formation process of extraordinarily high waves. The objective is to gain a deeper understanding of the formation process of freak waves in intermediate water depth such as at the location of the Draupner jacket platform where the "New Year Wave" occurred. The paper deals with the propagation of large amplitude breathers. It is shown that the mechanism of modulation instability also leads to extraordinarily high waves in limited water depth. Thereby different carrier wave length and steepnesses are systematically investigated to obtain conclusions on the influence of the water depth on the modulation instability and are accompanied by numerical simulations using a nonlinear potential solver.

INTRODUCTION

Freak wave observations and several accidents prove that the freak wave phenomenon is neither a seafarer tale nor an extreme rare event. There are many reports on accidents from cruise vessels encountering freak waves, such as the Louis Majesty (March 2010) and the Clelia II (December 2010) as well as on freak wave observations [1–4]. One of the most famous freak waves in history is the "New Year Wave" (NYW), an extraordinarily high single wave, recorded in the North Sea at the Draupner jacket platform on January 1st 1995 [5]. A general overview on real sea freak wave registrations and reported accidents can be found in [6].

The formation process of freak waves has been intensively investigated in the last decades. But since only single point registrations of real sea freak wave measurements are available, it is almost impossible to draw conclusions on the spatial development in front of and behind the measurement point. However, based on the measured responses of the Draupner platform published by [7], [8] assumed that the NYW occurred in a long crested sea
state and simulated the spatial development of the NYW using a model for weakly nonlinear spatial evolution of waves. This simulation revealed that the NYW “did not occur suddenly and unexpectedly”, due to the fact that the simulated NYW evolved from a large wave group 500 m in front of the target location. Also [9] simulated the spatial evolution of four freak waves recorded in the North Sea at the North Alwyn platform in 1997 [4], demonstrating that the lifetimes of the freak waves “vary from several seconds up to 42 s”, in which ”the freak waves travel up to 325 m”. Investigations in wave tanks are an inevitable element for a complete understanding of this phenomenon. [10] presented the evolution of an extreme wave at different positions in a wave tank, revealing that the extreme wave develops in less than half the wavelength from a relatively normal wave to an extreme crest. [11] reproduced the NYW in a seakeeping basin with successive measurements of the surface elevation of the extreme sea state along the tank – revealing freak wave occurrences at three different positions in the wave tank. These extreme waves are developing mainly from a wave group. The main question regarding the formation of the NYW in the tank and moreover regarding the real-sea formation process is the underlying mechanism. [12] estimated the Benjamin-Feir index of the NYW and stated that the abnormal wave is strongly nonlinear and showed that 7 waves are sufficient to manifest the modulation instability. Also [13] investigated the non-linearity and non-stationarity of the NYW using higher order time-frequency spectra to evaluate the nonlinearity. They also stated that the 7 waves leading to the NYW observed by [12] are enough for the evolution of the freak wave due to the modulational instability.

As aforementioned, the possible physical mechanisms of freak wave formation have already been identified - wave-current interaction, superposition of (nonlinear) independent component waves and/or modulation instability. The phenomenon of freak waves occurring due to the superposition of (nonlinear) independent component waves (spatial focussing) is well understood and explainable by Stokes wave theory [14] - in contrast to the sudden occurrence of exceptionally high waves. A substantial progress in the sense of the nonlinear wave-wave interaction leading to the sudden occurrence of freak waves is the discovery of the modulational instability of weakly nonlinear deepwater waves. [15] investigated the stability of periodic wave trains (Stokes waves) with small disturbances caused by a pair of side-band modes. This investigation reveals that weakly nonlinear deepwater wave trains are unstable to modulational perturbations due to the coupling through the nonlinear boundary conditions. The so called Benjamin-Feir instability results in a self-focussing effect of monochromatic waves, which causes a local exponential growth of the wave amplitude. The instability condition \( 0 < \delta \leq \sqrt{2k\zeta_0} \) has later been introduced as the Benjamin-Feir-Index (BFI), and a relation between spectral bandwidth \( \delta \) and wave steepness \( k\zeta_0 \) has been established by [16] and [17]. Further investigations show that the space and time evolution of weakly nonlinear deep

water wave trains obeys the nonlinear Schrödinger type equations [NLS] [18,19], which has been solved exactly by [20]. This solution predicts the existence of deep water wave envelope solitons and has been verified by experiments [21]. [22] introduced a fourth-order modified nonlinear Schrödinger (MNLS) equation for gravity waves and infinite water depth which was further refined by [23]. The results above are well understood, and robust from the physical [24] as well as the mathematical [25,26] point of view.

Even if the NLS equation is limited to weakly nonlinear water waves and narrow-banded spectra [27, 28] it features many of the dynamics of freak wave formation due to the self-focussing phenomenon [27]. The application of the NLS equation and the Benjamin-Feir instability to rogue wave crests and troughs (holes) in deep water wave trains has been shown by [29]. Furthermore it was shown by [30] that the NLS equation is applicable for the numerical investigation of the influence of the spectrum bandwidth on the occurrence of freak waves in random sea states using JONSWAP spectra with different Phillips parameters and enhancement factors. It has been shown theoretically that freak waves (\( H_{\text{max}} > 2.2H_t \)) occur more often in sea states with narrow bandwidth and higher enhancement factors respectively which are related to the modulation instability. This theoretical result has been verified qualitatively by experiments [31].

The focus of this paper lies on the formation process of a special type of freak waves – the so called breather solutions of the NLS equation. Therefore the Ma breather solution is applied for the generation of weakly nonlinear large amplitude waves in the seakeeping basin. The paper starts with a brief description of the theoretical background and setup. The main part presents the experimental and numerical results followed by the conclusions at the end.

**THE NLS EQUATION**

The NLS equation describes the evolution of the complex envelope \( A(x,t) \) whose relation to the surface elevation at the leading order is:

\[
\eta(x,t) = \frac{1}{2} \left[ A(x,t) \exp(i k_c x - \omega_c t) + \text{c.c.} \right]
\]

where \( \text{c.c.} \) stands for complex conjugate, \( k_c \) is the wave number of the carrier wave and \( \omega_c = \sqrt{gk_c \tanh(kd)} \) the angular frequency, with \( g \) the gravity acceleration. The NLS equation can be formally derived from the potential water wave equations under the hypothesis of small amplitudes and quasi-monochromatic waves; it takes the following form:

\[
\frac{\partial A}{\partial t} + c_g \frac{\partial A}{\partial x} + i \beta \frac{\partial^2 A}{\partial x^2} + i \gamma |A|^2 A = 0.
\]

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The second term takes into account the dispersive behavior of the surface elevation, while the last one is the nonlinear term. $C_g$ is the group velocity and the coefficient $\beta$ and $\gamma$ are a function of the carrier wave and frequency and depth. Their formal expression is given in the appendix in [32]. A remarkable property of such coefficients is that their ratio is larger than one for $k_c d > 1.36$ and becomes negative for longer waves. This changes drastically the physics described by the equation: for $k_c d > 1.36$ the equation has focussing properties, i.e. energy can be focalized through a nonlinear process that is the modulational instability. This mechanism is impossible for $k_c d < 1.36$.

Strictly speaking the modulational instability theory describes the growth of a small amplitude perturbation of a plane wave solution of the NLS equation; however nowadays it has become common to discuss about modulational instability also for large amplitude perturbations and not only for plane wave solution of the NLS.

Indeed, in the present paper we will discuss a particular solution of the NLS equation, which is known as Ma solution, and which corresponds to a plane wave with large amplitude perturbation. The reason of such choice is that ocean waves are characterized by wave packet and not by sinusoidal waves with small amplitude perturbations.

The NLS equation is suitable for describing the evolution in time of a space series. In wave tank facilities the situation is different because one provides the conditions at the wave maker and observe the evolution in space rather than in time. It is therefore useful to use the leading order approximation in NLS and exchange via an iterative procedure the derivative in space with the derivative in time. The resulting equation is:

$$\frac{\partial A}{\partial x} + \frac{1}{C_g} \frac{\partial A}{\partial t} + i \beta' \frac{\partial^2 A}{\partial t^2} + i \gamma' |A|^2 A = 0. \tag{3}$$

with

$$\beta' = \frac{\beta}{C_g}, \quad \gamma' = \frac{\gamma}{C_g} \tag{4}$$

Equation (3) will be named TNLS, where the $T$ stands for Time.

### The Ma solution of the TNLS equation

In order to sketch the derivation of the Ma solution of the NLS equation we follow the procedure described in [33]. The idea is to look for a solution of the TNLS in the form:

$$A(x,t) = A_0(x)[G(x,t) \exp(i\phi(x))] - 1 \tag{5}$$

where $G$, $\phi$ are real functions to be determined and $A_0 = a_0 \exp(-i \gamma' a_0^2 x)$. After some algebra and a particular choice of some extra function of time resulting from an integration in space, the following solution can be obtained:

$$A(x,t) = A_0(x) \left( \frac{-\sqrt{\mu^2} \cos(\gamma' a_0^2 \rho x) + i \sqrt{2} \rho \sin(\gamma' a_0^2 \rho x)}{\sqrt{2} \cos(\gamma' a_0^2 \rho x) - \sqrt{2 + \mu^2} \cosh(\mu t)} - 1 \right) \tag{6}$$

with

$$\mu = \frac{\mu}{a_0} \sqrt{\frac{B'}{\gamma'}}, \quad \rho = \sqrt{2 + \mu^2} \tag{7}$$

Equation (6) represents the Ma solution of the TNLS equation. Sometimes this is also called a breather solution because, in contrast to solitons which preserve their shape during their evolution, the Ma solution oscillates as it propagates. Note that such solution describes only the envelope of the wave and in order to build the surface elevation Eqn. (1) should be used and a carrier wavenumber $k_c$ should be selected.

### MA BREATER STUDIES

The above presented particular solution of the NLS (Ma breather) has been experimentally investigated. The experiments have been performed in the seakeeping basin of the Ocean Engineering Division of Technical University Berlin. The basin is 110 m long, with a measuring range of 90 m. The width is 8 m and the water depth is 1 m. On the one side an electrically driven piston type wave generator is installed. The wave generator is fully computer controlled and a software is implemented which enables the generation of regular waves, transient wave packages, deterministic irregular sea states with defined characteristics as well as tailored critical wave sequences. On the opposite side a wave damping slope is installed to suppress disturbing wave reflections.

The test setup consists of 10 surface piercing resistance-type wave probes installed in an interval of 5 m starting at 15 m and ending at 60 m in front of the wave maker.

### Experimental Results

Table 1 presents the main parameters of the investigated waves. The first column shows the investigated carrier periods and the second column the associated wave steepness $\varepsilon = k_c \zeta_m$. The wave steepness has been varied for each wave period to analyze its influence on the formation of the modulation instability. The carrier wave length $L_c = 2\pi/k_c$ (third column) has been chosen in such a way that the ratio between water depth $d$ and the carrier wave length $L_c$ fulfills the intermediate water depth condition ($d/L_c < 0.5$). The last column presents the product between carrier wave number and water depth which
instability. For $kd < 1.36$ the waves become stable as the nonlinear term in the NLS equation changes its sign [17].

Figure 1 illustrates a wave train of the investigated waves exemplarily. The sketch shows three Ma breathers in a row with the red curve marking the envelope of one breather. Two parameters have been determined for the following analysis – the maximum wave height and the maximum crest height for each breather (see Fig. 1). Since the investigated wave trains consist of several breathers in a row, the mean value has been evaluated for each registration. The maximum wave height as well as the asymmetry of the envelope curve indicates the modulation instability of the envelope – since increasing wave heights and in particular an increasing of the horizontal asymmetry of the envelope curve illustrates the energy transfer between the carrier and the sideband frequencies.

Figure 2 presents the wave and crest height development along the tank for the investigated waves. The diagram is arranged as follows: the three carrier wave periods are illustrated separately (black rectangles – top for $T_c = 1.25 \, s$, center for $T_c = 1.4 \, s$ and bottom for $T_c = 1.5 \, s$), the top diagram of each block presents the maximum wave height and the bottom diagram the maximum crest height of the breathers. Each diagram compares the results of the different initial steepnesses. It is obvious that the envelope of the breathers, i.e. the maximum wave and crest height (blue dots in Figure 2), for the smallest initial steepness ($\varepsilon = 0.1135$) do not change significantly during the development along the measuring section. This does not have to mean that the wave train is stable regarding the modulation instability but rather suggests that the distance the wave trains have to pass before the modulation instability leads to a significant energy transfer between the carrier frequency and the sidebands is much longer than the measuring section. This issue is discussed in detail in the subsequent subsection Numerical Investigations. Furthermore, Figure 2 reveals that with increasing initial steepness the breathers show an unstable behavior within the measuring section – an increase of the maximum wave height can be observed up to the last wave probe (red dots for $T_c = 1.25 \, s$ and $T_c = 1.4 \, s$ as well as green dots for $T_c = 1.5 \, s$). For the highest initial steepnesses (green dots for $T_c = 1.25 \, s$ and $T_c = 1.4 \, s$ as well as magenta dots for $T_c = 1.5 \, s$) the modulation instability leads to maximum wave heights as well as maximum wave crest heights within the measuring section (50 m for $T_c = 1.25 \, s$, 45 m for $T_c = 1.4 \, s$ and 60 m for $T_c = 1.5 \, s$). Table 2 illustrates the properties of these highest measured waves. It is obvious that the modulation instability leads to high steep waves with a large horizontal asymmetry. The maximum steepness values show that the highest wave within the breather reaches almost the maximum physically possible wave height/steepness at the measuring point since the maximum steepness reaches the wave breaking criterion \((k\zeta)_{\text{max}} = \pi \cdot 0.142 \cdot \tanh(kd) \approx 0.45 \tanh(kd))\) for the steepest wave. This corresponds to observations during the wave tank measurements, where plunging breakers have been observed for the highest waves. Please note that the observed wave breaking locations are not exactly identical with the positions of the wave probes, in particular for the run $T_c = 1.4 \, s$ with $\varepsilon_{\text{initial}} = 0.1702$

### TABLE 1. Overview on the investigated Ma breather solutions

<table>
<thead>
<tr>
<th>$T_c$ [s]</th>
<th>$k_c\zeta_a$</th>
<th>$d/L_c$</th>
<th>$k_c d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>0.1135</td>
<td>0.41</td>
<td>2.58</td>
</tr>
<tr>
<td>1.25</td>
<td>0.1362</td>
<td>0.41</td>
<td>2.58</td>
</tr>
<tr>
<td>1.25</td>
<td>0.1475</td>
<td>0.41</td>
<td>2.58</td>
</tr>
<tr>
<td>1.4</td>
<td>0.1135</td>
<td>0.33</td>
<td>2.05</td>
</tr>
<tr>
<td>1.4</td>
<td>0.1475</td>
<td>0.33</td>
<td>2.05</td>
</tr>
<tr>
<td>1.4</td>
<td>0.1702</td>
<td>0.33</td>
<td>2.05</td>
</tr>
<tr>
<td>1.5</td>
<td>0.1135</td>
<td>0.28</td>
<td>1.79</td>
</tr>
<tr>
<td>1.5</td>
<td>0.1475</td>
<td>0.28</td>
<td>1.79</td>
</tr>
<tr>
<td>1.5</td>
<td>0.1702</td>
<td>0.28</td>
<td>1.79</td>
</tr>
</tbody>
</table>

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**FIGURE 1.** EXAMPLE OF A MA BREACHER.
FIGURE 2. WAVE HEIGHT AND CREST HEIGHT DEVELOPMENT OF THE DIFFERENT MA BREATHERS ALONG THE TANK. THE THREE CARRIER WAVE PERIODS ARE ILLUSTRATED SEPARATELY (BLACK RECTANGLES – TOP FOR $\tau_c = 1.25$ s, CENTER FOR $\tau_c = 1.4$ s AND BOTTOM FOR $\tau_c = 1.5$ s), THE TOP DIAGRAM OF EACH BLOCK PRESENTS THE MAXIMUM WAVE HEIGHT AND THE BOTTOM DIAGRAM THE MAXIMUM CREST HEIGHT OF THE ENVELOPE. EACH DIAGRAM COMPARES THE RESULTS OF THE DIFFERENT INITIAL STEEPNESSES.
TABLE 2. PROPERTIES OF THE HIGHEST WAVES

<table>
<thead>
<tr>
<th>period</th>
<th>initial steepness</th>
<th>location</th>
<th>horizontal asymmetry</th>
<th>max. steepness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25 s</td>
<td>0.1475</td>
<td>50 m</td>
<td>0.64</td>
<td>0.438</td>
</tr>
<tr>
<td>1.4 s</td>
<td>0.1702</td>
<td>45 m</td>
<td>0.6</td>
<td>0.37</td>
</tr>
<tr>
<td>1.5 s</td>
<td>0.1816</td>
<td>60 m</td>
<td>0.65</td>
<td>0.436</td>
</tr>
</tbody>
</table>

the wave breaking location was at approximately 42 m.

Figure 3 exemplarily presents the formation of the highest waves for run $T_c = 1.25$ s with $\varepsilon_{initial} = 0.1475$ (see also Fig. 5), comparing the time series measured at the beginning of the measuring section (15 m) and the location of occurrence of the highest waves (50 m). It can be seen that during the evolution along the tank, the modulation instability leads to an energy transfer of the surrounding waves to the freak wave in particular in front of the freak wave. Figure 4 illustrates the corresponding amplitude spectra. The carrier frequency ($\omega_c \approx 5 \text{ rad/s}$) is clearly detectable for both registrations but more than half of the energy of the carrier wave is transferred to the sidebands at $x = 50$ m.

Numerical Investigations

As mentioned above, the experiments show that the modulation instability for large amplitude breathers can be detected in limited water depth but it is also identifiable that modulation instability appears only for higher initial steepnesses within the measuring section since the maximum wave and crest height (blue dots in Fig. 2) for the smallest initial steepnesses ($\varepsilon = 0.1135$) did not change significantly during the development along the measuring section. This denotes that either the breathers are stable or that the distance the wave trains have to pass before the modulation instability leads to a significant energy transfer between the carrier frequency and the sidebands is much longer than the measuring section.

To evaluate whether the aforementioned breathers are stable or unstable regarding the modulation instability the BF Index can be applied (the water depth limit $k_c d > 1.36$ for unstable waves is fulfilled – see Tab. 1). The instability condition $0 < \delta \leq \sqrt{2}k_c \zeta_a$ which has been obtained by standard linear analysis of the NLS equation assuming a small amplitude perturbation indicates the domain where a monochromatic wave – one carrier frequency and two small sideband frequencies – is unstable with respect to sideband perturbations. This condition has later been rearranged and introduced as the BFI [16, 17]:

$$BFI = \frac{\sqrt{2}\varepsilon}{\Delta K / k_0}$$ (8)
with \( \Delta K = 2 \cdot \Delta k \) and \( \Delta K = k_{0}/(2 \cdot N) \). But the wave sequences investigated in this paper are large amplitude breathers which comprise more than 2 sideband frequencies (cf. Fig. 4 – red curve) whereby the identification of the spectral band width \( \Delta K \) is hindered. However, it is assumed that the most significant influence on the modulation instability comes from the two sidebands close to the carrier frequency (cf. Fig. 4 – red curve). This assumption and Eqn. (8) results in a BFI of 1.2 for the smallest initial steepnesses (blue dots in Fig. 2) which suggests that these breathers are unstable too.

To prove if these breathers are also unstable to modulation instability, a nonlinear numerical wave tank is used to calculate the wave formation process for long distances. This potential theory solver (WAVETUB) has been developed at Technical University of Berlin for the simulation of nonlinear wave propagation \([34, 35]\). The two-dimensional nonlinear free surface flow problem is solved in time domain: the fluid is considered incompressible, and the flow is irrotational. The atmospheric pressure above the free surface is constant, and surface tension is neglected. Hence, the flow field can be described by a velocity potential which satisfies the Laplace equation for Neumann and Dirichlet boundary conditions. At each time step a new boundary-fitted mesh is created and the velocity potential is calculated in the entire fluid domain using the Finite Element method. On the basis of this solution the velocities at the free surface are determined by second-order differences. For long term simulations a numerical beach is implemented at the end of the wave tank by adding artificial damping terms to the kinematic and dynamic free surface boundary condition in order to suppress reflections. To develop the solution in time domain the fourth-order Runge-Kutta formula is applied. For the generation of the waves a moving wall is implemented at one side of the numerical wave tank, which enables the simulation of piston-type, flap-type and double flap-type wave boards. The wave board motion obtained in the physical wave tank can be directly used as input for the numerical calculations. The procedure is repeated until the desired time step is reached. A complete description of the numerical wave tank is published by \([35]\).

Firstly, WAVETUB is validated with the experimental results presented above. Run \( T_c = 1.25 \) s with \( \varepsilon = 0.1475 \) is chosen as a worst case regarding nonlinear wave propagation due to the high initial steepness and the registered freak waves at \( x = 50 \) m. Figure 5 compares the experimental results (blue curves) and the numerical calculations (red curves) at different positions in the tank up to the location where wave breaking occurs. The overall agreement is rather good. The agreement of the maximum wave heights at the bottom diagram (\( x = 50 \) m) differs slightly due to the fact that WAVETUB does not take into account wave breaking. It must be noted that the WAVETUB results have been shifted in time domain with a constant \( \Delta t = +0.2 \) s for a better comparability. The constant delay between the experimental results and the numerical calculations as well as the rather good overall agreement have been observed for all calculations. However, it is clearly identifiable that WAVETUB reproduces the formation process of nonlinear waves very accurately. Hence, it is possible to employ WAVETUB for the investigation of the breathers with the smallest initial steepness (\( \varepsilon = 0.1135 \)) to draw conclusion if and if so where the highest waves occur.

Figure 6 presents the numerical results which are separately illustrated for the three different carrier wave periods (black rectangles – top for \( T_c = 1.25 \) s, center for \( T_c = 1.4 \) s and bottom for \( T_c = 1.5 \) s). The top diagram of each block presents the surface elevation at \( x = 1 \) m in front of the wave board, the center diagram the surface elevation at \( x = 60 \) m and the bottom diagram the surface elevation at the location of the maximum wave height occurrence. The calculated surface elevations at \( x = 60 \) m are additionally compared to the experimental results to evaluate the quality of the numerical calculations. It is obvious that the waves with the smallest initial steepness are unstable as well.

Furthermore, Fig. 6 indicate a dependency between the distance, that a wave train needs to reach the maximum wave amplitude and the carrier wave length due to the limited water depth – the smaller the relative water depth \( d/L_c \), the longer is the distance the wave train needs to reach the maximum wave amplitude for the same BFI. This conclusion is supported by the experimental results since the initial steepnesses have been more increased for the longer carrier wave length (cf. Fig. 2) to obtain the maximum wave amplitude within the measuring section.

To evaluate the water depth dependency on the distance the wave train has to pass to reach the maximum wave height, numerical calculations with WAVETUB for different water depths for the \( T_c = 1.4 \) s breather with the smallest initial steepness (\( \varepsilon = 0.1135 \)) have been performed. Therefore the original wave board motion has to be modified to generate identical wave sequences at different water depths as the physical piston type wave maker as well as the numerical one covers the full water depth. This implies that for the numerical calculations the height of the moving wall increases with increasing water depth. The numerical wave board motion for the different water depths has been adjusted using the Biesel function \([36]\), which relates the wave board stroke to the wave amplitude at the position of the wave maker in dependency of the water depth and geometry of the wave maker. To assess the accuracy of the numerically reproduced wave sequences for the different water depths – in particular as the initial steepness plays a major role for the formation of the highest wave – registrations (for the additional water depths) at \( x = 10 \) m in front of the wave maker are compared to the target registration \( d = 1 \) m in frequency domain. The distance of \( x = 10 \) m is chosen to ensure at one hand that the generated wave is completely developed and on the other hand that the location is close to the wave maker so that the spectrum of the wave sequence does not change significantly due to the modulation instability. Fig. 7 presents the comparison of the spectra for the different water depths (\( d = 0.75 \) m...1 m...1.5 m...2 m...2.5 m...4 m).
FIGURE 5. EXPERIMENTS vs. WAVETUB – COMPARISON BETWEEN EXPERIMENTAL RESULTS (BLUE CURVES) AND NUMERICAL CALCULATIONS (RED CURVES) AT DIFFERENT POSITIONS IN THE BASIN UP TO THE LOCATION WHERE THE WAVE BREAKS ($x = 50$ m).
FIGURE 6. NUMERICAL RESULTS FOR THE THREE CARRIER FREQUENCIES ($T = 1.25$ s TOP BLACK RECTANGLE, $T = 1.4$ s CENTRAL BLACK RECTANGLE AND $T = 1.5$ s BOTTOM BLACK RECTANGLE) FOR THE SMALLEST INITIAL STEEPNESS $\varepsilon = 0.1135$. THE TOP DIAGRAMS OF EACH BLOCK PRESENT THE REGISTRATION AT $X = 1$ m IN FRONT OF THE WAVE BOARD, THE CENTER DIAGRAMS COMPARES THE CALCULATED SURFACE ELEVATIONS AND THE EXPERIMENTS AT $X = 60$ m AND THE BOTTOM DIAGRAMS PRESENT THE REGISTRATION AT THE LOCATION OF THE MAXIMUM WAVE HEIGHT.
It is obvious that the agreement between the different wave spectra for the different water depths is excellent. From this comparison, it can be deduced that the wave sequences generated for the different investigated water depths are identical.

Figure 8 presents surface elevation snapshots at the time of occurrence of the first maximum wave height for the investigated water depths - the water depth increases from top to bottom. The initial and the termination ramp of the wave sequence are excluded in the evaluations, i.e. the three breathers inside the wave sequence are considered. For the determination of the location of the maximum wave height occurrence, the maximum temporary crest amplitude has been tracked during the spatial development along the numerical wave tank up to the position where the wave crest reaches its maximum and breaks. In this context it should be noted again that the wave breaking phenomenon cannot be simulated using WAVETUB and that the simulation of the steepest waves prior to the breaking point is delicate - a smooth function for the FEM grid is implemented to avoid a program termination due to wave breaking which influences the wave profile and propagation after the wave breaking event. However, the quantitative detection of the location of the maximum wave height is unaffected.

Figure 8 clearly demonstrates that the distance the envelope has to pass before reaching its maximum wave height is strongly depending on the water depth. In intermediate water depth ($d \leq 1.5$ m) the distance increases significantly with decreasing water depth and approaches infinity probably at $k_c d < 1.36$ where the waves become stable as the nonlinear term in the NLS equation changes its sign [17].

CONCLUSIONS

This paper presents an experimental study on the evolution of weakly nonlinear large amplitude waves to gain a deeper understanding on the formation process of freak waves. The focus lies on the formation of freak waves in intermediate water depths such as at the location of the Draupner jacket platform where the NYW occurred. For the investigations, a particular solution of the NLS equation – the Ma (breather) solution – has been applied to produce large amplitude waves in the seakeeping basin of Technical University Berlin, where the surface elevation has been measured at different positions along the basin. Thereby different carrier wave length and initial steepnesses are systematically investigated.

It is shown that the mechanism of modulation instability also leads to extraordinarily high waves in intermediate water depths – the formation of freak waves up to the maximum physical possible wave height has been observed (see Tab. 2). It has been identified that the distance the wave trains have to pass before the modulation instability leads to a significant energy transfer between the carrier frequency and the sidebands depends on the initial steepness of the breather as predicted by the theory: the higher the initial steepness the shorter the distances the wave train has to pass to reach the maximum wave height. In this context, it has been demonstrated that the nonlinear numerical wave tank WAVETUB is applicable for the simulation of nonlinear wave propagation, in particular for the simulation of long distances and varying water depths which cannot be covered by test facilities. The experiments as well as the numerical simulations reveal that limited water depth influences the formation process significantly. In intermediate water depth the distance the wave train has to pass to reach the maximum wave height increases significantly with decreasing water depth and approaches infinity probably at $k_c d < 1.36$.

In general, the investigations reveal that the breather solutions are applicable for the generation of steep wave events for wave-structure investigations in test facilities.

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FIGURE 8. NUMERICAL RESULTS FOR THE $T_c = 1.4 \text{s}$ BREATHER WITH THE SMALLEST INITIAL STEEPNESS ($\varepsilon = 0.1135$): SURFACE ELEVATION SNAPSHOTS AT THE TIME OF OCCURRENCE OF THE FIRST MAXIMUM WAVE HEIGHT FOR THE INVESTIGATED WATER DEPTHS ($d = 0.75 \text{ m}..1 \text{ m}..1.5 \text{ m}..2 \text{ m}..2.5 \text{ m}..4 \text{ m}$) – THE WATER DEPTH INCREASES FROM TOP TO BOTTOM.
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