LONGITUDINAL FORCES AND BENDING MOMENTS OF A FPSO

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ABSTRACT

For the design of FPSOs the vertical bending moment is a key parameter to ensure safe operation. If analyzed at water line level, however, the unknown influence of longitudinal forces may distort the results. Hence, a segmented FPSO model with midship force transducers at two levels is investigated in various deterministic wave sequences to identify the vertical bending moment and its associated neutral axis as well as the superimposing longitudinal forces. It is shown that the neutral axis is far below the water line level, with the consequence, that extreme cyclic loads at deck level would be expected. However, as the associated longitudinal forces - even if significant - generate a counteracting moment, this effect is largely compensated.

Both, frequency- and time-domain results are presented. With frequency-domain analysis the profound data for the standard assessment of structures, concerning seakeeping behaviour, operational limitations and fatigue are obtained. In addition, time-domain analysis in real rogue waves gives indispensable data on extremes, i.e. motions and structural forces.

INTRODUCTION

Ship losses such as the sinking of the single hull tankers Erika (1999, Fig. 1) and Prestige (2002, Fig. 2) as well as of the double-hull tanker Ievoli Sun (2000, Fig. 3) illustrate that structural failures in heavy seas are still a major problem.

A key issue is the vertical bending moment. According to the IACS-Common Rules [1] the classification societies are considering a ship in design waves, and distinguish between hogging and sagging condition. The associated vertical wave bending moments

\[ M_{WV} = L^2 \cdot B \cdot c_{WV} \cdot c_1 \cdot c_L \cdot c_M \]

(1)

depend on:

- \( c_{WV} \) - wave coefficient

\[ c_{WV} = \left[ 10.75 - \left( \frac{300 - L}{100} \right)^{1.5} \right] \cdot \frac{1}{c_{RF}} \]

Fig. 1: Erika sinking (source: dpa)
• $c_{1H}$ - hogging condition coefficient:
  \[ c_{1H} = 0.19 \cdot c_B \]
• $c_{1S}$ - sagging condition coefficient:
  \[ c_{1S} = -0.11 \cdot (c_B + 0.7) \]
• $c_L$ – lengths coefficient:
  \[ c_L = 1 \]
• $c_M$ - distribution factor:
  \[ c_{MH} = 1 \text{ (hogging)} \]
  \[ c_{MS} = 1 \text{ (sagging, } v_0 = 0 \text{ m/s)} \]

with ship length $L$ (= $L_{pp}$), width $B$ and block coefficient $c_B$.

A more sophisticated approach requires the investigation of ship response in irregular seas. This paper focuses on experimental and numerical analysis of a FPSO-vessel, and presents results in frequency and time domain. In particular, ship loads in tailored rogue waves are studied.

**EXPERIMENTAL PROGRAM**

For the experimental investigations a wooden model of the selected FPSO design has been built at a scale $\lambda = 1:81$ (Fig. 4). The model is cut into three segments with intersections at $L_{pp}/4$ and $L_{pp}/2$ measured from bow.

![Fig. 4: Segmented FPSO – equipped with connecting force transducers at different levels](image)

The main dimensions of the FPSO are $L_{pp} = 259.90$ m, $B = 46$ m, draft $D = 16.67$ m, displacement $\n = 174000$ t and block coefficient of $c_B = 0.87$. This model – with the segments connected by transverse beams for measuring the bending moments – has already been used in earlier investigations [2,3,4]. In this case, numerical and experimental results of the vertical bending moments correspond quite well if related to the waterline level. Measured at deck level, however, significant differences are observed. To clarify these discrepancies, similar seakeeping tests are performed in this project, however, the model segments are now connected by force transducers at two vertical levels to exactly determine the superposition of bending moments and associated longitudinal forces (Fig. 4).

As a hypothesis it is assumed that the FPSO experiences significant horizontal forces which generate a counteracting bending moment and ease the loads at deck level. (Fig. 5)

Seakeeping tests are performed in head seas. As a typical sequence of an extreme wave, the New Year Wave [5] has also been simulated numerically and at model scale (Fig. 6)

To simulate such rogue wave sequences experimentally the target position $x_{\text{target}}$ of the deterministic wave/structure interaction is selected. At this location, the target wave train is either specifically
This wave train is transformed upstream to the position of the wave maker which requires an adequate wave propagation model [6]. On the basis of linear wave theory the specified amplitude distribution of the target wave train is given as Fourier transform \( F(\omega, x_{\text{target}}) \) with circular frequency \( \omega \) as a function of wave number \( k \). Adaptation of the phase spectrum to the wave maker location \( x_0 \) gives the Fourier transform in \( x_0 \):

\[
F(\omega, x_0) = F(\omega, x_{\text{target}}) e^{i \omega t - k(x_{\text{target}} - x_0)}. \tag{2}
\]

For reducing the number of time steps until the wave maker starts to operate, the wave train is shifted by time \( t_{\text{shift}} \), i.e.

\[
F(\omega, x_0) \cdot e^{i \omega t_{\text{shift}}}. \tag{3}
\]

For extremely high wave trains like the New Year Wave, however, the associated wave propagation is modelled by a semi-empirical nonlinear method which is based on the fact that short and high wave groups with strong nonlinear characteristics evolve from long and low wave groups which are characterized by linear principles [7], [8].

From the wave train at the position of the wave maker the corresponding control signals are calculated using the hydrodynamic transfer function (relating wave board motion to wave elevation), the geometrical RAO (which considers the flap type) and the electric-hydraulic RAO. This control signal is used to generate the specific wave train which is measured at the target position in the tank.

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Fig. 8: Wave sequence (New Year Wave) and related forces at midship. Based on these data the vertical bending moments and its associated neutral axis as well as the longitudinal forces and the related (counteracting) bending moment (related to waterline and deck level) are presented at full scale.
Comparison of bending moments at different levels of registration

moments at waterline level
moments at deck level
moments at neutral axis

Fig. 9: Midship-Bending moments at different vertical levels

NUMERICAL SIMULATION

Various numerical codes have been developed to analyse motions and loads of hydrodynamically compact structures in frequency and time domain. In the following we present short descriptions of WAMIT, SEAWAY and the IST-code, and introduce our time-domain code F2T+ which calculates the vertical bending moments (as well as motions etc.) as function of arbitrary wave sequences.

WAMIT

For the evaluation of motions, forces and bending moments the program system WAMIT (Wave Analysis, developed at Massachusetts Institute of Technology) for wave/structure interaction at zero-speed [9] is applied.

Hydrodynamic analysis

The analysis of a compact rigid body with six degrees of freedom is described by a boundary value problem. The total velocity potential \( \phi(x,t) \) for an inviscid, incompressible fluid and irrotational flow follows from Laplace equation

\[
\Delta \phi(x,t) = 0 .
\]  

Assuming linear theory the total velocity potential \( \phi \) is given as superposition of the individual potentials due to incoming plane waves and the wave systems arising from the motions of the body

\[
\phi(x,t) = \phi_0 + \sum_{k=1}^{6} \phi_k
\]  

with \( \phi_0 \) incident wave potential

\( \phi_1 \) potential of scatter wave field

\( \phi_k \) potential of the radiation wave field evoked by a motion in mode \( k \).

These potentials describe the initial wave field \( \phi_0 \) and its reflection on the body surface resulting in the scatter wave field \( \phi_1 \). The last term in Eq. (5) describes the radiation wave fields \( \phi_k \) which follow from body motions in 6 degrees of freedom.

On the wetted body surface normal velocity is zero. For an oscillating body this condition results into:

\[
\frac{\partial \phi_\tau}{\partial n} = \xi \cdot n^*, \text{ on } S_b .
\]  

The linearized kinematic and dynamic boundary conditions on the free surface are merged into the generalized free surface condition:

\[
\frac{\partial \phi_j}{\partial z} - g^2 \phi_j = 0 .
\]  

On the ocean bottom normal velocities are zero:

\[
\frac{\partial \phi_j}{\partial n} = 0 , \text{ for } z = -d .
\]  

\( \phi_k \) (\( i=0,1,...,7 \)) holds for any potential, which are superimposed to form the complete solution. Finally in the far field the Sommerfeld radiation condition for the scatter and radiation wave field must be satisfied:

\[
\lim_{\tau \to \infty} \sqrt{R} \left( \frac{\partial \phi_j}{\partial R} - ik \phi_j \right) = 0 , \quad j=1,...,7 .
\]  

The initial boundary value problem, defined by Laplace equation (4) and the above boundary conditions, is transformed into integral equations by applying Green’s second theorem [10] and, after some manipulation, we obtain:

\[
2\pi \phi + \iint_{S_b} \left( \frac{\partial}{\partial n} G_{n0} - G_{0n} \right) dS + \int_{-\infty}^{t} \iint_{S_b} \left( \partial G_{tn} - G_{nt} \right) dS d\tau = 0 .
\]  

This equation allows solving for the unknown scatter and radiation potentials on the mean position of the body surface \( S_b \) in time-domain. In frequency-domain the second integral term vanishes to zero. The wetted body surface has to be discretized into \( N \) panels (Fig. 10) where the boundary conditions are satisfied on the respective collocation points. Based on this potential the instationary Bernoulli equation gives the (linearized) dynamic pressure:

\[
p_{\text{dyn}} = -\rho \frac{\partial \phi}{\partial t} ,
\]  

which defines the forces and moments acting on the body:

\[
\mathcal{F} = \int_{S_b} p_{\text{dyn}} n^* dS .
\]  

With Newton’s second law and assuming the body and its forcing comprise a stable linear system, the equation of motion is obtained [11]:

\[
(\ddot{M} + a^{\varepsilon}_{\xi} \dot{z} + B_{\xi} + C_{\xi} + \int_{-\infty}^{t} \dddot{K}(t-\tau) \dot{z}(\tau) d\tau) = \mathcal{F}(t) .
\]  

In frequency-domain, Eq. (13) is solved for harmonic excitation. Divided by the wave amplitude \( \zeta_a \) the equation is reduced to:
with the unknown response amplitude operator forcing.

of the motion in frequency-domain.

bending moment of the ship: line, multiplied with the flexural stiffness results in the results from complex addition of the individual motions. For several bending modes, the total deflection indices are reserved for the conventional rigid body deformations can be calculated. Legendre polynomials ship bending modes the associated structural polynomial.

The indexing takes into account, that the first 6 indices are reserved for the conventional rigid body motions. For several bending modes, the total deflection results from complex addition of the individual deflection lines. Twice differentiation of the deflection line, multiplied with the flexural stiffness results in the bending moment of the ship:

\[ M_b(x) = -w''(x) \cdot EI_y(x) \]  

The above WAMIT procedure has been successfully used to analyze vertical bending moments of stationary ships with high block coefficient [14].

**SEAWAY**

The program SEAWAY is a frequency-domain ship motion code, based on linear strip theory, to calculate the wave induced loads, motions, added resistance and internal loads for six degrees of freedom of displacement ships, in regular and irregular waves. For calculating the vertical bending moments the solid mass distribution is given to model the actual load case of the model. SEAWAY calculates the vertical bending moments from contributions of vertical forces as well as horizontal forces (see Theoretical Manual of Strip Theory Program “SEAWAY for Windows” [15]). Thus, SEAWAY can calculate the vertical bending moments with respect to an arbitrary vertical reference frame.

**IST-code**

The IST calculations are done using a time-domain nonlinear strip theory code. The method assumes that the nonlinear contribution for the vertical bending moment is dominated by hydrostatic and Froude-Krylov forces, thus these components depend on the instantaneous hull wetted surface. Radiation and diffraction forces are linear. Additionally green water loads on the deck, which contribute to the calculation of motions and global loads, are represented by the momentum method. A detailed presentation of the method is given by Fonseca and Guedes Soares [16], [17].

The exciting forces due to the incident waves are decomposed into a diffraction part and the Froude-Krylov part. The diffraction part, which is related to the scattering of the incident wave field due to the presence of the moving ship, is kept linear. Since this is a linear problem and the exciting waves are known a priori, it can be solved in the frequency-domain and the resulting transfer functions be used to generate a time history of the diffraction heave force and pitch moment. The Froude-Krylov part is related to the incident wave potential and results from the integration at each time step of the associated pressure over the wetted surface of the hull under the undisturbed wave profile.

The hydrostatic forces and moments are calculated at each time step by integration of the hydrostatic pressure over the wetted hull under the undisturbed wave profile. The radiation forces, which are calculated using a strip method, are represented in the time-domain by infinite frequency added masses, radiation restoring coefficients (which are zero for the zero speed case), and convolution integrals of memory functions. The convolution integrals represent the effects of the whole past history of the motion accounting for the memory effects due to the radiated waves. This code has been validated against measurements for the case of a containership [18] and of an FPSO hull [19] showing good results.

The vertical forces associated with the green water on deck, which occurs when the relative motion is larger than the free board, are calculated using the momentum method [20]. The mass of water on the deck is proportional to the height of water on the deck, which is given by the difference between the relative motion and the free board of the ship. This approach to include these forces has been validated in [21].

The methodology to produce the correct wave field and the associated wave exciting forces which are consistent with a deterministic wave record (measured at one point fixed in space) is presented in [22].

**F2T+ code**

With frequency-domain results the motion behaviour of arbitrary structures in waves are investigated very fast and efficiently. The derived results, however, can be interpreted only statistically. If cause-reaction effects are
of interest and wave/structure interaction are evaluated in detail a time-domain analysis in deterministic wave trains is required. By Fourier-transforming frequency-domain results into impulse response functions and subsequent convolution with arbitrary wave trains a simple method is given to investigate wave-structure interactions in time-domain [23,24,25]. The response amplitude operators calculated by WAMIT are transformed into impulse response functions by Fourier transformation (Fig. 11) [11]:

$$K_i(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_i(\omega)e^{i\omega t} d\omega$$

(17)

For this purpose a Fortran routine F2T by J.N. Newman has been provided to the authors as beta-version. The Fourier Transforms are evaluated by Filon numerical integration. With known impulse-response functions $K_i(t)$ of the motions the time-dependent response in arbitrary wave trains $\zeta(t)$ are calculated by convolution:

$$s_i(t) = \int_{-\infty}^{\infty} K_i(t-\tau)\zeta(\tau)d\tau$$

(18)

This improved F2T+ procedure combines the

- Transformation of the complex frequency-domain RAOs into time-domain impulse-response functions
- Convolution of the impulse response function with arbitrary wave sequences to determine the behaviour of a structure in time-domain [23].

Verification of the presented method $F2T+$ is carried out for single structures with TiMIT (Time Domain Analysis Massachusetts Institute of Technology, [26]) which is based on the same theory as WAMIT.

In the following applications the wave trains measured during experiments are used to carry out the simulations and allow therefore direct comparison between simulation and experiment.

**FREQUENCY-DOMAIN RESULTS**

Fig. 12 presents the transfer functions of the vertical bending moment at midship comparing numerical programs to experimental results. Only WAMIT gives the same peak frequency as the experiments, whereas IST-code and SEAWAY yield slightly higher frequencies. Concerning the peak values, the model test data show good agreement with WAMIT and IST-code whereas SEAWAY predicts slightly lower values.

As SEAWAY is the only program system that can account for the vertical position of the connecting elements at the model the sensitivity of this parameter is also investigated. Fig. 13 presents a comparison of numerical and experimental results of the transfer function of the vertical bending moments related to a reference frame at deck level. As compared to the RAO-results at waterline level the maximum values match very good with SEAWAY, however, the results of previous investigations [2], [3] are slightly lower.
**TIME-DOMAIN RESULTS**

Fig. 14 and 15 present F2T+ results compared to experimental data. The vertical bending moment in Fig. 14 is related to the waterline level. The agreement is excellent. The corresponding longitudinal forces (Fig. 15) show some discrepancies by phase and amplitude which are related to the fact that the distance of the two force transducers is too small.

In conclusion it is stated that the magnitude of the longitudinal forces is quite significant. As the position of the neutral axis is far below the waterline level these forces induce an additional moment which is antiphase to the vertical bending moment. As a consequence, cyclic loads at the waterline level and especially at deck level are significantly reduced by the action of longitudinal forces.

Finally it will be useful to compare the results in frequency- and time-domain with design bending moments as calculated by Eq. (1). With a wave coefficient \( c_{WV} = 10.5 \), the hogging and sagging condition coefficient \( c_{1H} = 0.1653 \) and \( c_{1S} = -0.1727 \), respectively, the design wave bending moment amounts to

\[
M_{WV-hogging} = 5.4 \cdot 10^6 \text{ kNm} \\
M_{WV-sagging} = -5.6 \cdot 10^6 \text{ kNm}
\]

Thus, the 25.6 m freak wave (Fig. 14) which causes a bending moment of about \( 4 \cdot 10^6 \text{ MNm} \) is well covered by IACS-rules.

Similar results follow from the standard procedure for determining the significant (and maximum) vertical bending moment as a function of the sea state characteristics. As shown in Fig. 16 [2,3] a variety of standard spectra (Pierson-Moskowitz – normalized for \( H_S = 1 \text{ m} \)) is multiplied by the squared RAO of the
midship bending moment (related to waterline level – see Fig. 12), and the resulting response spectra are evaluated to obtain the significant bending moment (double amplitude) as function of the zero-up-crossing period. Highest values of the significant RAO of bending moment (double amplitude) are expected in the range of $T_0 = 11$ s (Fig. 16 bottom diagram). Even at this worst case ($T_0 = 11$ s) – assuming a significant wave height of $H_S = 15$ m (with a most probable maximum wave height of $H_{\text{max}} = 1.86 \cdot 15$ m = 28 m within a 3 hour storm) – the significant bending moment yields $M_{\text{sign}} = 3 \cdot 10^6$ kNm with a most probable maximum amplitude of $M_{\text{WV, max}} = 5.6 \cdot 10^6$ kNm which is still below IACS–limitations.

Note that Fig. 16 also includes the (smoothed) energy spectrum of the New Year Wave with a zero-up-crossing period of $T_0 = 10.8$ s (see Fig. 11). Thus, this sea state ($H_S = 11.92$ m, $H_{\text{max}} = 25.6$ m) is quite critical.

CONCLUSIONS AND OUTLOOK

This paper presents a comprehensive study of the vertical bending moments of a FPSO due to rogue wave impact comparing numerical simulations and seakeeping model tests. A freak wave sequence registered in the North Sea has been generated at model scale using nonlinear methods. All model tests are performed with a segmented FPSO for measuring the vertical midship bending moment by

- two force transducers on each side at different vertical positions or
- an instrumented transverse beam alternatively at waterline level or deck level.

It is shown that the neutral axis is far below the waterline level, with the consequence, that extreme cyclic loads at deck level would be expected. However, as the associated longitudinal forces are quite significant and generate a counteracting moment, this effect is largely compensated.

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