Multi-body Systems in Waves - Impact of Hydrodynamic Coupling on Motions

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ABSTRACT: Offshore installation procedures frequently require the operation of two or more structures in close proximity. Due to radiated and diffracted waves these multi-body systems are hydrodynamically coupled. As a consequence significant additional motions are observed. In case of lifting operations limitations are given by significant or maximum allowable relative motions which follow from the rigid body motions of each structure. With frequency-domain analysis the motion behaviour in harmonic waves is determined very fast and efficiently, and the resulting response amplitude operators (RAO) are used to derive operational limitations depending on seastate. Such results however have statistical character; it is not possible to derive cause-reaction chains or predict maximum motions in a deterministic wave train.

With investigations in time-domain the cause-reaction chain of a multi-body system in a wave sequence can be analysed in detail and the associated hydrodynamic coupling is quantified. As direct time-domain calculations are time consuming a method of transforming frequency- into time-domain results has been developed. This method is based on impulse response functions which are derived from the complex RAOs by Fourier transformation. Once the impulse response functions are known, the response of a structure in arbitrary wave trains can be determined by convolution. This method takes advantage of the fast frequency-domain analysis for specific time-domain investigations in deterministic wave trains, including hydrodynamic coupling as well as the influence of memory effects. With this method the interaction of hydrodynamically coupled structures in arbitrary wave sequences can be analysed in time-domain. In particular extreme situations in predefined wave trains are identified.

The procedure of transforming frequency-domain results into time-domain is rarely applied in naval architecture and ocean engineering despite of its simplicity and effectiveness. As a special feature the influence of the hydrodynamic coupling can be studied in time-domain and compared to frequency-domain results.

1 INTRODUCTION

The term multi-body systems is used in a very restricted manner considering two or more rigid structures operating in close proximity. One single structure floating in an undisturbed wave field is excited to motions. Additional wave fields are generated, the radiated waves, which are superimposed to the initial waves. Simultaneously the initial waves are scattered by the structure. The radiated and scattered waves are treated as disturbance of the initial wave field resulting in a three-dimensional complicated wave field. A second structure in vicinity is exposed to this superimposed wave field and generates its own scatter and radiation wave pattern. The resulting motions differ significantly from the motion behaviour of the single structure. The interaction between the structures is called hydrodynamic coupling. When treating hydrodynamic compact structures viscous effects can be neglected and the problem of moving structures can be solved using potential theory leading to a boundary value problem.

During operations involving several structures one of the most important limiting criteria is given by the (vertical) relative motion between the structures (e.g. lifting operations). Relative motions are derived from the rigid body motions which are strongly affected by hydrodynamic coupling. In frequency-domain, the motion behaviour is given by response amplitude operators, describing the motion amplitude and phase in harmonic waves. In a natural seaway, obtained by linear superposition of harmonic wave components covering the relevant frequency range, the response is determined by spectral analysis. The resulting response spectrum describes the energy density of the motions. Required data are derived from the moments of the spectrum in analogy to the significant wave height.
and the zero-upcrossing period. So far, the analysis of multi-body systems is restricted to frequency-domain, assuming linear system behaviour and linear wave theory. Time-domain calculations so far are carried out to include nonlinearities, e.g. mooring characteristics, viscous damping, roll motion or wave theories of higher order (Chakrabarti 2001). This paper deals with the effect of hydrodynamic coupling on motions in time-domain. A method of transforming results from frequency- to time-domain is described. The Fourier transformation of the response amplitude operators results in the impulse response functions. They describe the response of a system to a unit impulsive forcing. This transformation is based on a Fortran routine (F2T as postprocessor to WAMIT), developed by J.N. Newman. Once the impulse response functions are known, the time-dependent motions in arbitrary wave trains can be determined by convolution.

The transformation method and results in frequency- as well in time-domain are presented on the example of a lift operation of the crane semisubmersible Thialf (operated by Heerema Marine Contractors) and a transport barge. Three stationary states of a load transfer (10000t) are investigated:

1. The transport barge, carrying the load, is positioned under the crane hook (Fig. 1). During this state no mechanical connections occur: The structures are coupled hydrodynamically. The radii of gyration and the center of gravity considers the load on the barge. The semisubmersible is equipped with a rapid-ballast system (Grafoner 1989), with filled ballast tanks under the cranes and empty tanks in the front columns.

2. The load is fastened to the crane hook and the rope is pretensioned to 80\% (static case) of the load mass. During this state early lift-off of the load as well as a slack rope should be avoided. It is the starting point for the rapid-ballast system. The pretension of the rope leads to a mechanical coupling of the structures. Within given limits this coupling is treated as rigid connection, i.e. no relative motions between the structures are allowed.

3. In the third phase the load is hanging on the hook. System parameters include the activated rapid-ballast system (Fig. 1). With the rapid-ballast system the load is lifted by 4.5m in 90s.

Special attention in this investigation is paid to the vertical relative motion between the structures, since this is a limiting criteria of lift operations. Horizontal relative motions can be analysed in analogy with the presented method. But they are in general affected by the influence of mooring or dynamic positioning systems, which are not considered in this analysis and therefore results for horizontal relative motions are not presented in this paper.

2 THEORETICAL BACKGROUND
In this section only a short theoretical overview is given. The equations represent a guide through the complicated theory.

2.1 Mathematical description
For the determination of forces and motions of freely floating bodies in waves diffraction theory is applied. With the assumption of inviscid and incompressible fluid and irrotational flow the flow field around the bodies is described by velocity potentials. The boundary value problem is governed by Laplace’s equation:

\[
\Delta \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0, \tag{1}
\]

with \(\Phi\) as total velocity potential. Since linear wave theory is used, this velocity potential is composed by superposition of the potentials from initial, radiation and scatter wave fields. The following equation sums
up the wave fields, arising from M bodies:
\[
\Phi = \phi_0 + \sum_{k=0}^{M+M} \phi_k + \sum_{j=1}^{6M} \phi_j.
\] (2)

In this equation the potential of the initial wave field is designated as \(\phi_0\), while \(\phi_k\), \(k = 1, 2, \ldots, M\) designates the scatter potentials. Each of the M bodies performs motions in six rigid body modes, each related to a radiation potential \(\phi_j\).

In addition, the boundary conditions on the surrounding boundaries (free surface, ocean bottom, body surface and infinity) have to be satisfied (Clauss & Birk 1994).

The solution of the boundary value problem is based on Green’s second theorem. The introduced Green function is known for the problem of a floating body in waves (Wehausen & Laitone 1960). After exploiting the boundary conditions the remaining integral equation consists uniquely of the integral over the surface of the body, since the Green function satisfies the Laplace equation (1) and the boundary conditions except the condition on the body surface (Mei 1989):
\[
2\pi \phi + \int_{(S_b)} \phi \frac{\partial G}{\partial n} dS = \int_{(S_s)} G \frac{\partial \phi}{\partial n} dS.
\] (3)

This equation is implemented in the diffraction code WAMIT (Wave Analysis Massachusetts Institute of Technology, (Lee 1995; Department of Ocean Engineering, MIT 1994)). The body surface is discretised to solve the system of integral equations numerically.

To overcome accuracy problems due to the gap between the structures resulting in hydrodynamic resonance in the fluid-domain a direct solver is used to solve the system of equations, instead of the faster iterative solver which has converging problems. The problem of irregular frequencies (concerning mainly the barge at relatively shallow draft) is avoided by a high number of panels and by using the option of WAMIT to remove the effects of irregular frequencies.

2.2 Hydrodynamic analysis

After evaluating the total potential (Eq. 2) the pressure is derived from the linearised, instationary Bernoulli equation:
\[
p = -\rho g z - \rho \frac{\partial \Phi}{\partial t}.
\] (4)

The forces and moments acting on the body are determined by integrating the pressure over the body surface:
\[
F = \int_{(S_b)} p n^* dS = F_{stat} + F_{dyn}.
\] (5)

In this equation hydrostatic as well as hydrodynamic forces and moments are included (Birk 1998). Related to Newton’s second law:
\[
M \cdot \ddot{\mathbf{z}} = F_{stat} + F_{dyn},
\] (6)

the equation of motion for a stable, linear system is obtained:
\[
(M + A) \cdot \ddot{\mathbf{z}} + B \cdot \dot{\mathbf{z}} + C \cdot \mathbf{z} = F_{err}.
\] (7)

By assuming harmonic exciting forces and motions in harmonic waves \(\zeta(t) = \zeta_a e^{-i\omega t}\):
\[
F_{err}(t) = F_{err, a} \cdot e^{-i\omega t + i\gamma},
\] (8)
\[
\dot{\mathbf{g}}(t) = \zeta_a \cdot e^{-i\omega t + i\gamma},
\] (9)
\[
\mathbf{g}(t) = -i\omega \zeta_a \cdot e^{-i\omega t + i\gamma} = -i\omega \mathbf{g}(t),
\] (10)
\[
\ddot{\mathbf{g}}(t) = -\omega^2 \zeta_a \cdot e^{-i\omega t + i\gamma} = -\omega^2 \ddot{\mathbf{g}}(t),
\] (11)

the equation of motion becomes time-independent
\[
\begin{align*}
\{-\omega^2(M + A) - i\omega B + C\} \ddot{\mathbf{z}}_a e^{i\gamma} = & F_{err, a} e^{i\gamma}. \end{align*}
\] (12)

The associated response amplitude operator \(H(\omega) = \frac{\ddot{\mathbf{z}}_a e^{i\omega t}}{\zeta_a e^{i\omega t}}\) is obtained by relating motions and forces to the wave amplitude \(\zeta_a\). The required hydrodynamic coefficients (added mass \(a_{kl}\) and potential damping \(b_{kl}\)) are derived from the radiation potentials (Newman 1977):
\[
a_{kl} + \frac{i}{\omega} b_{kl} = \rho \int_{(S_s)} \varphi_l n^*_k dS.
\] (13)

The response amplitude operators \(H(\omega)\), as solution of the motion equation (12), describe amplitude and phase of the motions of the structures in harmonic waves. The motion behaviour in a natural seaway is obtained by spectral analysis. Relevant characteristics, as significant double amplitude and zero-upcrossing period of the response are derived from the associated response spectrum. These values are of statistical nature. Note that maximum values are dependent on the number of motion cycles (for derivation see section 2.4).

2.3 Spectral analysis

The response amplitude operators obtained from the solution of the motion equation (12) hold as first criteria to assess the motion behavior of a structure. The decision, however, if an operation is feasible needs information about the behaviour in natural seaways. A random, time-dependent wave train can be treated as superposition of many harmonic wave components.
The specific energy of such an elementary wave is proportional to its quadratic amplitude:

\[ S_c(\omega_i)d\omega = \frac{1}{2}\zeta_{ai}^2. \]  

(14)

With all wave components the energy density spectrum is formed. For engineering purposes standard spectra as the Pierson-Moskowitz spectrum are formulated (see e.g. (Clauss et al. 1994)). In analogy the energy of the response is derived from the motion amplitude \( s_{ai} = s_a(\omega_i)\zeta_{ai} \), i.e.:

\[ S_s(\omega_i)d\omega = \frac{1}{2}s_{ai}^2 = \frac{1}{2}\left( \frac{s_a(\omega_i)}{\zeta_{ai}} \right)^2 \zeta_{ai}^2. \]  

(15)

By substituting the wave amplitude \( \zeta_{ai} \) from Eq. (14) the energy of the response becomes:

\[ S_s(\omega_i) = \left( \frac{s_a}{\zeta_{ai}} \right)^2 S_c(\omega_i), \]  

(16)

respectively the response spectrum:

\[ S_s(\omega) = |H(\omega)|^2S_c(\omega). \]  

(17)

From sea and response spectrum characteristic values are derived by area and moments. Significant wave height and zero-upcrossing period are obtained from the sea spectrum:

\[ H_s = 4\sqrt{m_0} \quad \text{and} \quad T_0 = 2\pi\sqrt{\frac{m_0}{m_2}}, \]  

(18)

while significant double amplitude and zero-upcrossing period of the response are given by the response spectrum:

\[ (2s_a)_s = 4\sqrt{m_{0,s}} \quad \text{and} \quad T_{0,s} = 2\pi\sqrt{\frac{m_{0,s}}{m_{2,s}}}, \]  

(19)

with:

\[ m_i = \int_0^\infty \omega^4S(\omega)d\omega = \begin{cases} m_0 : \text{area} \\ m_2 : \text{2nd moment.} \end{cases} \]  

(20)

Table 1: Comparison of maximum wave heights with the first approximation (Eq. (32)), the most probable (Eq. (29)) and the mean maximum wave height (Eq. (31)) for different numbers of waves \( N_w \).

<table>
<thead>
<tr>
<th>( N_w )</th>
<th>10</th>
<th>100</th>
<th>10³</th>
<th>10⁵</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{H_{\text{max}}}{H_s} )</td>
<td>1.07</td>
<td>1.52</td>
<td>1.86</td>
<td>2.40</td>
</tr>
<tr>
<td>( \frac{H_{\text{max}}}{H_o} )</td>
<td>1.12</td>
<td>1.54</td>
<td>1.87</td>
<td>2.40</td>
</tr>
<tr>
<td>( \frac{H_{\text{max}}}{H_r} )</td>
<td>1.21</td>
<td>1.61</td>
<td>1.94</td>
<td>2.46</td>
</tr>
<tr>
<td>( \Delta % )</td>
<td>13.1</td>
<td>5.92</td>
<td>4.3</td>
<td>2.5</td>
</tr>
</tbody>
</table>

To derive assessment criteria in arbitrary seaways the above described procedure is repeated for several seastates with varied zero-upcrossing periods \( T_0 \). The resulting significant double amplitudes are normalized by the significant wave height. This leads to the significant response amplitude operator as a function of the zero-upcrossing period:

\[ \frac{(2s_a)_s}{H_s} = f(T_0), \]  

(21)

which allows the comparison of different structures or configurations and serves as basis for operation decisions.

2.4 Estimation of maximum values

The derivation of maximum values is sketched on the example of wave heights (Jacobsen 2005). The results can be adopted to obtain maximum double amplitudes of the response. The distribution of the wave heights in a registration is approximated by a Rayleigh distribution with the assumption that the random process is narrow banded as well as stationary and ergodic:

\[ \varphi_R(H) = \frac{4H}{H_s^2}e^{-\frac{2H^2}{H_s^2}}. \]  

(22)

With the wave height distribution the probability of occurence results in:

\[ \Phi_R(H_1) = \int_0^{H_1} \varphi_R(H)dH = 1 - e^{-\frac{2H_1^2}{H_s^2}}, \]  

(23)

and the related probability of exceedence is:

\[ P(H > H_1) = 1 - \Phi_R(H_1) = e^{-\frac{2H_1^2}{H_s^2}}. \]  

(24)

Given an extremely long registration the probability of exceedence for one arbitrarily selected wave is
Figure 3: Transformation (F2T+) of frequency-domain results (complex RAOs) into time-domain on the example of the heave motion of the transport barge in a high wave sequence (New Year Wave).

The probability of exceeding the given niveau of the $H_1$ of two succeeding waves is slightly higher: $P(H > H_1) = 1 - \Phi(H_1)$. Selecting $N_w$ succeeding waves, the probability that one of the $N_w$ waves is higher than $H_1$ is:

$$P(H > H_1) = 1 - \Phi^{N_w}(H_1).$$

The associated probability of occurrence is:

$$\Phi_{H_{\text{max}}}(H_1) = \Phi^{N_w}(H_1) = \left[1 - e^{-\frac{2H_1^2}{H_s^2}}\right]^{N_w}. \quad (26)$$

Differentiation of (26) with wave height leads to the probability density function of the maximum wave height:

$$\varphi_{H_{\text{max}}}(H) = N_w \left[1 - e^{-\frac{2H^2}{H_s^2}}\right]^{N_w-1} \frac{4H - \frac{2H^3}{H_s^2}}{H_s^3} e^{-\frac{2H^2}{H_s^2}}. \quad (27)$$

Note that for $N_w = 1$ the density function is reduced to the underlying Rayleigh distribution. This density function is plotted in Figure 2 for different numbers of waves $N_w = 10, 10^3, 10^5$. In addition the associated distribution of the maximum wave height (Eq. (26)) is shown. As a result of this distributions it can be concluded, that wave heights greater $H_{\text{max}}/H_s = 2$ are exceeded by more than 28% in a 3-h-seastate with approximately 1000 waves (see Eq. (25)).

The most probable maximum wave height $\bar{H}_{\text{max}}$ (the wave height with the highest probability) as a function of the number of waves is derived from the maximum of the density function (27):

$$d\varphi_{H_{\text{max}}}(H) \frac{d}{dH} = 0. \quad (28)$$

This leads to an equation for the number of waves $N_w$:

$$N_w = \frac{H_s^2}{4\bar{H}_{\text{max}}^2} + \frac{1}{4\bar{H}_{\text{max}}^2} - \frac{H_s^2}{4\bar{H}_{\text{max}}^2} e^{-\frac{3\bar{H}_{\text{max}}^2}{H_s^2}}. \quad (29)$$

Alternatively, the mean maximum wave height $\bar{H}_{\text{max}}$ corresponds to the center of area of the density function:

$$\bar{H}_{\text{max}} = \int_0^\infty H\varphi_{H_{\text{max}}}(H)dH. \quad (30)$$
Figure 6: Response amplitude operators of heave and pitch motion for SSCV Thialf in waves from $\beta = 180^\circ$, and heave motion of barge in waves from $\beta = 0$ and $180^\circ$ with and without hydrodynamic coupling.

Figure 7: Wave fields in a harmonic wave ($\omega = 0.8 \text{rad/s}$) around the single barge as well as around the multi-body system for two wave headings $\beta = 0^\circ$ and $\beta = 180^\circ$.

This equation is solved numerically. For Gauss distributed random processes an approximation is given by (Longuet-Higgins 1952):

$$\frac{H_{\text{max}}}{H_a} = \sqrt{\frac{\ln(N_w)}{2}} + \frac{0.57722}{\sqrt{8 \ln(N_w)}}. \quad (31)$$

In this equation the known and common approximation

$$\frac{H_{\text{max}}}{H_a} = \sqrt{\frac{\ln(N_w)}{2}} \quad (32)$$

is contained. Table 1 summarizes and compares the solutions of the different approximations of the maximum wave height. Note that the mean maximum wave height gives higher values for all numbers of waves $N_w$. The above discussion illustrates that Freak waves (see e.g. (Haver 2000; Wolfram et al. 2000; Faulkner 2003)) are not really rare events.

2.5 Transformation to time-domain

With frequency-domain results the motion behaviour of multi-body systems in waves are investigated very fast and efficiently. The derived results, however, can be interpreted only statistically. If cause-reaction effects are of interest and wave/structure interaction are evaluated in detail a time-domain analysis in deterministic wave trains is required. A simple method of transforming frequency-domain results into time-domain enables investigation of hydrodynamically coupled structures including memory effects (Cummins 1962). The response amplitude operators calculated by WAMIT are transformed into impulse response functions by Fourier transformation:

$$K_i(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_i(\omega)e^{i\omega t}d\omega. \quad (33)$$

For this purpose a Fortran routine F2T by J.N. Newman has been provided to the authors as beta-version. The Fourier Transforms are evaluated by Filon numerical integration. With known impulse-response functions of the motions $K_i(t)$ the time-dependent response in arbitrary wave trains $\zeta(t)$ are calculated by convolution:

$$s_i(t) = \int_{-\infty}^{\infty} K_i(t - \tau)\zeta(\tau)d\tau. \quad (34)$$

Verification of the presented method is carried out for single structures with TiMIT (Time Domain Analysis
Massachusetts Institute of Technology, (Korsmeyer et al. 1999)) which is based on the same theory as WAMIT but without the possibility of investigating multi-body systems. In Figure 4 time-dependent heave motions of the crane semisubmersible *Thialf* as well as the barge are presented. Especially the heave motions of the barge agree very well. Figure 5 compares RAOs calculated with $F2T+$ in harmonic waves (wave periods from 3 to 10 s and some characteristic frequencies) to WAMIT results.

3 RESULTS IN FREQUENCY-DOMAIN

In Figure 6 the response amplitude operators of heave and pitch motion of the crane vessel (SSCV) as well as the heave motion of the barge are shown. The influence of the hydrodynamic coupling affects mainly the motions of the barge. Two wave headings are plotted:

- $\beta = 0^\circ$ i.e. the barge is located on the lee side of the crane vessel. In this case the barge benefits of the shield effect - the resulting motion behaviour is very favourable compared to the single structure.
- $\beta = 180^\circ$ i.e. the barge is located on the weather side and exposed to initial as well as diffracted and radiated wave fields. Strong interaction effects are observed.

These results illustrates that the consideration of hydrodynamic coupling is indispensable for the determination of safe operations. In Figure 7 the wave fields about the barge as single structure (at left), about the multi-body system with barge on the lee side of the semisubmersible (middle) and the barge on the weather side (right) are shown in a harmonic wave ($\omega = 0.8 \text{ rad/s}$). The effect of hydrodynamic coupling, i.e. the disturbed wave field influencing the barge motions is evident.

From the rigid body motions, i.e. heave and pitch of the semisubmersible as well as heave of the barge the response amplitude operator of the relative motion between the crane hook and the barge is determined by complexe addition in order to include correctly phase angles. The resulting RAOs for wave headings $\beta = 0^\circ$ and $180^\circ$ are presented in Figure 8 for the third lift state. At lower frequencies the resonance motions of the semisubmersible dominate the relative motions while at higher frequencies (i.e. at frequencies where operations are feasible) the motions of the barge are relevant.

Table 2: Maximum double amplitudes for $N_w = 20, 200, 1000$ waves associated with $N_A$ cycles of motion ($H_s = 2m$, $T_0 = 5s$).

<table>
<thead>
<tr>
<th>$N_w$</th>
<th>20</th>
<th>200</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_A$</td>
<td>11</td>
<td>112</td>
<td>562</td>
</tr>
</tbody>
</table>

| $[2s_{rel3,H3}]_{max}$ $\text{[m]}$ | 1.23  | 1.63 | 1.86 |
| $[2s_{rel3,H3}]$ $\text{[m]}$ | 0.64  | 0.85 | 0.97 |

Especially in the case of the barge positioned at the lee side ($\beta = 0^\circ$) the deviation is significant. When the operation involves two structures with significantly different displacements a configuration of the smaller vessel at the lee side of the bigger if obviously favorable.

Based on these frequency-domain results maximum double amplitudes of the response are calculated from Eq. (31) in a sea state with a significant wave height of $H_s = 2m$ and a zero-upcrossing period of $T_0 = 5s$. This is considered as limit sea state for lift operations. Table 2 presents the resulting maximum double amplitudes for different numbers of waves $N_w$. The corresponding numbers of motion cycles $N_A$ are determined from $N_A = N_wT_0/T_{0,rel}$. The zero-upcrossing period of the relative motion is obtained from Eq. (19).

The required significant double amplitudes in this sea state during the third lift state are derived from Figure 9:

- $\beta = 0^\circ$: $(2s_{rel})_s = 0.52m$
- $\beta = 180^\circ$: $(2s_{rel})_s = 1.68m$

The resulting values in Table 2 show clearly that the estimated maximum double amplitudes are sensible.
and compares time-domain (TD) and frequency-domain (FD) results. Both, the significant and maximum relative motions at all lift states, respectively. A lift assisted by a rapid-ballast system (e.g. with 200 waves).

Based on the frequency-domain results we restrict the presentation of time-domain results to the case:
- significant wave height \( H_s = 2\text{m} \)
- zero-upcrossing period \( T_0 = 5\text{s} \)
- wave heading (barge on lee side) \( \beta = 0^\circ \)

Figure 10 shows a related wave registration as well as the associated relative motions during the third lift state. As the entire registration includes 1000 waves we can compare time-domain and frequency-domain data. The maximum wave height with \( H_{\text{max}} = 3.78\text{m} \) is slightly smaller than the estimated maximum wave height (3.88m from Eq. (31)). Table 3 presents significant and maximum relative motions at all lift states, and compares time-domain (TD) and frequency-domain (FD) results. Both, the significant and the maximum double amplitudes are considerably smaller than expected from frequency-domain.

This surprising phenomenon is analysed by investigating the relative vertical motion due to different wave group characteristics. In Figure 10 three sectors are marked. In the first sequence from 2650-2750s the smallest relative motion occurs, while in the second the maximum relative motion is observed. The third includes the maximum wave height. The zoomed ranges with the associated relative motions are shown in Figure 11. They illustrate the different motion behaviour:

- In the first sequence very small motions are observed. The maximum double amplitude of the relative motion with \( (2s_{\text{rel}})_{\text{max}} = 0.23\text{m} \) (third lift phase) is quite lower than the value estimated in frequency-domain with 20 waves (see Table 2) and it is even smaller than the significant double amplitude of the entire registration. The zero-upcrossing period \( (T_{0,\text{rel}} = 6.4\text{s}) \) of the relative motion is very low. The aim of a safe lift operation is to start the rapid-ballast system in such a sequence with low motion.

- The second sequence is characterised by the maximum double amplitude of the relative motion. Though the exciting wave heights are quite similar to the waves of the first sequence (most of them are smaller than the significant wave height) the response is quite different. The associated zero-upcrossing period of the relative motions with 8.2s is high. Such events during lifting operations should be avoided.

- During the third sequence, containing the highest wave, the response is something between the first and the second sequence. The maximum double amplitude is 0.44m, not really spectacular if we remember that this amplitude is caused by the maximum wave. The zero-upcrossing period is \( T_{0,\text{rel}} = 7.13\text{s} \), located between the two others.

![Figure 9](image.png)

**Figure 9:** Significant response amplitude operators of vertical relative motion calculated with and without hydrodynamic coupling at three wave headings for the third lift state (see Fig. 1).

Table 3: Significant and maximum double amplitudes of vertical relative motion in a wave sequence with \( H_s = 2\text{m} \) and \( T_0 = 5\text{s} \) comparing time- and frequency-domain results (LS: lift state, TD: time-domain, FD: frequency-domain).

<table>
<thead>
<tr>
<th>Lift State</th>
<th>((2s_{\text{rel}})_{\text{LS}}) [m]</th>
<th>((2s_{\text{rel}})_{\text{max}}) [m]</th>
<th>(T_{0,\text{rel}}) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS 1</td>
<td>0.37</td>
<td>0.65</td>
<td>8.5</td>
</tr>
<tr>
<td>LS 2</td>
<td>0.33</td>
<td>0.67</td>
<td>8.9</td>
</tr>
<tr>
<td>LS 3</td>
<td>0.34</td>
<td>0.58</td>
<td>8.1</td>
</tr>
</tbody>
</table>

Apparentley, the maximum wave excites a rather unspectacular reaction, while waves of medium height cause high or low responses due to different group characteristics of the exciting waves. The investigation of a single (random) 1000-wave-sample is evidently not sufficient as the joint probability of wave height and group characteristic is of crucial importance for the response. As shown in Figure 2 a single 1000-wave-sequence may contain a maximum wave height between \( 1.6H_s < H_{\text{max}} < 2.5H_s \) with maximum probability of \( H_{\text{max}} = 1.86H_s \). This observation, however, is focused on wave height only. If waves are embedded in a resonance-dominated wave sequence the response is expected to be quite significant (Fig. 11 - detail 2). Consequently, time-domain...
investigations require a greater number of simulated registrations to arrive at realistic estimates of extremes. Alternatively, the extreme wave group may be stretched or compressed to obtain critical joint occurrences of high waves and resonant motion behaviour as has been shown by (Fonseca et al. 2005). Apparently, the selected wave sequence of Figure 10 results in quite a favourable motion behaviour in time domain (see Table 3).

The presented results proof, that frequency-domain analysis is inevitable for the determination of limits. In addition time-domain analysis provides information which is quite helpful for operation decisions because they yield relations between wave and structure behaviour.

5 CONCLUSIONS

In this paper a transformation method from frequency- to time-domain is presented. Response amplitude operators from frequency-domain are converted into impulse response functions by Fourier transformation. This procedure allows the determination of the time-dependent responses in arbitrary wave sequences. Critical situations can be analysed in detail in time-domain. The method combines the advantages of the fast and efficient frequency-domain analysis with the option of analysing cause-reaction phenomena in time-domain.

The method is applied on the example of the crane semisubmersible Thialf and a transport barge. The impact of hydrodynamic coupling is studied in frequency- as well as in time-domain. Detailed analysis of the structure response show that wave frequency is a major parameter. If a wave sequence contains a wave packet with resonance components it may be far more dangerous then a single high wave.

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