TAILOR MADE FREAK WAVES WITHIN IRREGULAR SEAS

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ABSTRACT

The paper presents a nonlinear technique for generating high and highest freak waves by using deterministic wave packets which are also embedded into irregular seas.

First, the technique is developed for small water elevations with a linear description of the wave. Afterwards the method is extended to a nonlinear description based on the introduction of a “wave information” which includes the (linear) information of waves as Fourier Transforms. Combined with a modified wave celerity (information speed) the propagation of high waves can be developed by using Stokes higher order solutions or semi empirical expansion equations. As a result, the nonlinear wave contours, registrations and associated wave data are derived from the respective wave information.

As the mentioned methods are not only used as a theoretical approach for describing steep wave trains a wide range of applications for generating high deterministic waves in model basins is presented. Thus extreme waves with heights of more than 3 m have been generated. This technique has also been adapted for the demands of computer controlled seakeeping tests. As both model and wave probe do not necessarily move at the same velocity, a time domain procedure is introduced for transforming the registration of the wave elevation from the wave probe position to the model position which leads, especially in astern and quartering seas, to extraordinary wave contours experienced by a moving vessel.

NOMENCLATURE

\[ |F(\omega_j, x_c)| \] Fourier spectrum 
\[ F(\omega_j, x_l) \] Fourier transform 
\[ I(t, x) \] wave information 
\[ W \] Fourier transform of linear vertical particle velocity 
\[ a(t) \] wave envelope 
\[ c(\omega, d) \] phase velocity 
\[ c_0 \] wave celerity of regular wave 
\[ d \] water depth 
\[ g \] acceleration due to gravity 
\[ i, j, k, l, n \] indices 
\[ \Delta k \] wave number step 
\[ k_j \] wave number 
\[ p_{kj}, q_{kj} \] frequency related coefficients for calculation in time domain 
\[ t \] time 
\[ t_c \] concentration point (time) 
\[ t_{end} \] end of time interval 
\[ t_i = t_0 + i\Delta t \] 
\[ t_{shift} \] time shift 
\[ t_0 \] start of time interval, in general \[ t_0 = 0 \] 
\[ u \] horizontal velocity 
\[ \bar{u}_i \] horizontal velocity 
\[ v_M(t_i) \] model velocity 
\[ w \] vertical velocity 
\[ w_a \] amplitude of vertical velocity 
\[ x \] coordinate in space 
\[ \bar{x} \] horizontal time averaged particle displacement
\[ \Delta x \text{ space step} \]
\[ x_c \text{ concentration point} \]
\[ x_l \text{ discrete location} \]
\[ x_0 \text{ starting point} \]
\[ z \text{ coordinate in space} \]
\[ z_0 \text{ starting point} \]
\[ \alpha \text{ horizontal particle displacement according to linear wave theory} \]
\[ \gamma \text{ phase, vertical particle displacement according to linear wave theory} \]
\[ \delta \text{ rudder angle} \]
\[ \delta_{ij} \text{ Kronecker symbol} \]
\[ \zeta(t, x_l, z) \text{ surface elevation, vertical particle displacement} \]
\[ \zeta_{aw} \text{ wave amplitude, maximum surface elevation} \]
\[ \zeta_1 \text{ surface elevation of a linear wave packet} \]
\[ \theta \text{ phase} \]
\[ \phi \text{ course angle} \]
\[ \mu \text{ roll motion, velocity potential} \]
\[ \phi \text{ encounter angle, phase} \]
\[ \phi(\omega_j, x_c) \text{ phase spectrum} \]
\[ \Delta \omega \text{ frequency step} \]
\[ \omega_{ej} \text{ encounter frequency} \]
\[ \omega_j \text{ circular frequency } \omega_j = j\Delta \omega \]

INTRODUCTION

Extreme wave conditions in a 100-years design storm arise from the most unfavorable superposition of component waves of the related severe sea spectrum — a coincidence which requires a very extended test duration.

As an alternative the component waves of a project oriented design spectrum are tentatively generated in such a sequence that all waves superimpose without phase shift at a given position. This option is based on spectral analysis which describes the apparently chaotic wave field of a natural seaway as the superposition of an infinite number of independent harmonic waves with amplitude \( \zeta_{aw} \) and random phase \( \gamma_0 \)

\[
\zeta(t) = \sum_{n=1}^{m} \zeta_{aw} \cos(\omega_n t - \gamma_0). \tag{1}
\]

Each component wave contributes an amount of energy to the seaway proportional to its squared wave amplitude (Clauss et al., 1992). The most unfavorable event under storm conditions is the in-phase superposition of component waves in the seaway. As these “freak” waves are possible events during the life-cycle of a marine structure it may be selected as the relevant design wave.

Of course, we expect high non-linearities. Therefore the scope of this paper covers the following topics:

- Design waves are the in-phase superposition of several component waves of the most severe sea spectrum (not necessarily all component waves): this rare event can be simulated intentionally in seakeeping tests to investigate maximum loads. Please note that the maximum wave height might also be limited due to breaking phenomena caused by limited water depth and/or wave length.
- The genesis of design waves is a highly nonlinear process. However, as these short and high wave groups evolve from long and low (linear) wave groups, the nonlinear characteristics can be derived from linear principles. Consequently, extremely high “freak” waves have a “linear (describable) past”.
- These converging design waves can be used for the experimental generation of extreme waves within a storm sea.

Our simulation technique is based on a computer controlled nonlinear procedure which governs the generation of a short, specifically tailored wave train with a “design” spectrum. As the wave train is exactly defined in space and time it is easy to transpose its registration to any position along the tank.

This paper proposes a method for the description of the shape variation of nonlinear transient wave packets during its propagation. Short and high wave groups with strong nonlinear characteristics evolve from long and low wave groups which are described by linear principles. Based on this fact, the idea was born that the linear complex Fourier transform (surface velocity) of the long and low wave group could be used as the characteristic information of the wave train. Consequently, all nonlinear wave elevations, velocities and accelerations should be derivable from this linear wave information. Also the transformation of the wave train registration from one position to any other position is feasible by using the linear wave information. During its metamorphosis the total energy of the transient wave is invariant, if breaking phenomena are excluded.

- At first, the paper describes the generation of linear wave trains and its linear transformation to selected positions. This is an important prerequisite as the same procedure will be used for the transformation of the linear wave information to establish a nonlinear wave train. In this case the linear wave celerity as a function of the wave frequency is replaced by a nonlinear “information” celerity which depends on frequency, space, and time.
- The next section introduces the development of the numerical nonlinear description of the entire wave field of the transient wave train by integrating the mutually dependent particle motion equations in time domain and by introducing empirical terms. Furthermore, common nonlinear methods are used to adapt the empirical coefficients to the application purpose.
- Finally, the paper presents model test results from computer controlled capsizing tests where the wave excitation at the
ship model has to be related to the motions of the ship model. High wave groups are embedded within the model sea which is a realization of an ITTC storm spectrum.

**TRANSIENT WAVE TECHNIQUE FOR SEAKEEPI NG TESTS**

Transient waves for model excitation were originally proposed by [3] and further developed by [4], [5]. [3] recommended a special type of transient waves, i.e. Gaussian wave packets which have the advantage that their propagation behavior can be predicted analytically ([6]).

With increasing efficiency and capacity of computers the restriction to a Gaussian distribution of wave amplitudes has been abandoned, and the entire process is performed numerically ([7]). The shape and width of the wave spectrum can be selected individually for providing sufficient energy in the relevant frequency range. As a result the wave train is predictable at any stationary or moving location. In addition, wave orbital motions as well as pressure distribution and the vector fields of velocity and acceleration can be calculated. According to its high accuracy the technique is capable of generating special purpose transient waves:

- defined wave groups for seakeeping tests;
- very high “freak” waves;
- regular wave trains with embedded high wave groups
- storm seas with steep wave groups occurring at defined positions in time and space.

Even complicated interactions of wave group and structure are easy to analyze:

- With regard to wave filter systems the reflected and transmitted wave train can be separated from the initial wave packet.
- The transformation to arbitrary - stationary or moving - positions or models allows a clear relation of cause and effect.

**Linear Wave Generation**

The synthesis and up-stream transformation of wave packets is developed from its so-called concentration point. At this position all waves are superimposed without phase shift resulting in a single high wave peak. Starting from this concentration point, the Fourier transform is transposed to the selected interaction position of structure and wave train, and finally to the position of the wave generator.

On the basis of linear wave theory the desired amplitude distribution of the wave packet is given as Fourier spectrum $F(\omega_j, x_c)$. Together with the related phase spectrum at the concentration point $x_c$, $\varphi(\omega_j, x_c)$ this gives the unscaled Fourier transform at $x_c$:

$$ F(\omega_j, x_c) = |F(\omega_j, x_c)| e^{i\varphi(\omega_j, x_c)}, \varphi = \omega_j t_l - k_j x_c $$

with circular frequency $\omega_j = j \Delta \omega$ as a function of wave number $k_j$, $t_l = i \Delta t$. After scaling the wave train due to the desired maximum wave elevation at the target position we obtain the related Fourier transform $F(\omega_j, x_t)$ at $x_t$. Adaptation of the phase spectrum to the target location $x_t$ gives the Fourier transform in $x_t$:

$$ F(\omega_j, x_t) = |F(\omega_j, x_t)| e^{i(\omega_j t_l - k_j (x_t - x_c))}. $$

In order to reduce the number of time steps until the wave maker starts to operate the wave train is shifted by time $t_{shift}$:

$$ \tilde{F}(\omega_j, x_0) = F(\omega_j, x_0) e^{-i\omega_j t_{shift}}. $$

Finally, this Fourier transform is multiplied by the hydraulic RAO relating main board motion to wave elevation, the geometrical RAO, and the hydraulic RAO for each wave paddle.

Fig. 1 shows a wave train as a function of time at different positions (water depth $d=4.20m$) and the related amplitude spectrum of the complex Fourier transform. The wave train can be transformed from a function of time $a(t, x_0)$ at a fixed location $x_0$ to a function of space $a(x, t_0)$ at a given time $t_0$ (see Fig. 1). The transformation of a wave train, $\tilde{\zeta}(t, x_c) \Rightarrow \zeta(t_c, x)$, starts from the concentration point $(t_c, x_c)$ where the phase spectra both in time and space domain are identical. First, the wave numbers $k_j$ are deduced from the Fourier spectrum’s discrete circular frequencies $\omega_j = j \Delta \omega$ by numerical solution of the dispersion equation:

$$ \omega_j^2 = k_j g \tanh(k_j d). $$

Values $k_j$ are assigned to values of $|F(\omega_j)|$. However, these values are wrongly scaled since points are transformed from equidistant $\omega_j$-scale to non-equidistant $k_j$-scale. Correct scaling follows from congruence of the area under the spectrum function:

$$ |F(k_j)| = |F(\omega_j)| \frac{\Delta \omega}{\Delta k_j}. $$

Interpolation and inverse Fourier transformation give the transient wave packet as a function of space. Note that this transformation can be done without loosing any information of the transient wave train.

**Calculation of Wave Trains at Stationary and Moving Locations**

For converting measured surface elevations to arbitrary points in time and space, three cases are considered:
1. Let the registration of a transient wave packet $\zeta(t_i,x_l)$ be given at the wave tank location $x_l$. The wave train at another fixed location $x_l + k$, $k \in \mathbb{Z}$, is calculated by shifting the Fourier transform's phase by $k(x_l + k - x_l)$:

$$\zeta(t_i,x_l + k) = \frac{1}{2\pi} \sum_j F(\omega_j,x_l)e^{i(\omega_j t_i - k(x_l + k - x_l))} \Delta \omega.$$  

(7)

2. Let the registration be taken under constant velocity $v_M = \text{const.}$ and encounter angle $\phi$ against the wave train. Thus the real wave numbers can be calculated from the encounter frequencies $\omega_j$ by solving

$$\omega_j = \omega_{ej} + k(\omega_j) \cdot v_M \cdot \cos \phi$$  

(8)

numerically. Then the above procedure is repeated.

3. For capsizing tests the undisturbed surface elevation at a ship’s reference point is required. As this wave elevation cannot be measured directly it is calculated from a registration at another (fixed or constantly moving) location using the following scheme:

$$\omega_j = \omega_{ej} + k(\omega_j) \cdot v_M(t_i) \cdot \cos \phi(t_i)$$  

(9)

where $\Delta x_i$ stands for the time varying distance between both locations.

Considering well-defined time windows during the test and excluding disturbances within the main signal, the described techniques can be applied to all types of (linear) model seas.

**Nonlinear Transient Wave Description**

Two types of high and dangerous waves are observed:

- high and steep "walls of water", which travel at high speed and exist for quite a long time. These waves are modelled by soliton theory ([9]).
- spontaneous interaction of wave groups, mostly within a storm, which can become very high and steep. This dangerous phenomenon – stable for a very short time – can be modelled by wave packets ([10]).

For high wave packets the above described procedures based on linear wave theory cannot be used. Fig. 2 shows the comparison of low and high wave trains, measured at two different
positions. It illustrates that a high wave train is slightly faster when propagating. Note that all three wave trains are generated by synchronous wave maker motions, which have been calculated by linear theory. The diagram at the right hand at a position $x_2=84.95m$ illustrates that the entire high wave packet is about 0.73s faster. The illustration also visualizes the existence of lower waves following the wave train. This phenomenon is a consequence of incorrect wave maker motions which have been calculated by linear theory. To allow the accurate generation and analysis of high waves an improved procedure has to be used.

For regular waves nonlinear solutions have been introduced by Gerstner (1802), Stokes (1880), and Crapper (1957). Recent solutions for irregular seas have been proposed by [11], [12] and [13]. None of these methods is applicable to the analysis of converging wave groups which become shorter, higher, and therefore faster during their way through the channel.

[14] and [15] have presented the analysis of extreme waves using the Creamer transform. [16] presented an outstanding experimental study on focusing waves. The target of this paper is to describe the idea of a wave generation technique which allows a fast and variable application of different types of model seas. In contrast to many sophisticated methods to describe and simulate extreme wave groups with defined parameters as for example in [17] the methods explained here allow the backward calculation of wave maker control signals for the simulation of arbitrary wave trains in a wave tank - like the New Year Wave Shown in Fig. 6.

**Empirical Description Based on Linear Wave Theory.** The method proposed in this paper is based on the fact that short and high wave groups having strong nonlinear characteristics evolve from long and low wave groups which are basically characterized by linear principles. As the total energy of the transient wave is invariant during its metamorphosis, the initial linear Fourier transform of the surface velocity is selected as the backbone of "wave information". Based on this "wave information" the shape variation of the linear transient wave train during propagation is calculated. At selected positions a nonlinear expansion is accomplished by integrating the mutually dependent particle motion equations in time domain, resulting in a numerical nonlinear description of the transient wave train as a function of time or space at any fixed or moving reference point. After the nonlinear wave train has passed the concentration point, the primordial linear Fourier spectrum of the long, low wave group can be found again which can be used to monitor the accuracy of the transformation.

Our numerical method confirms experimental observation that nonlinear high-amplitude waves

- are slightly faster than small waves, and therefore also slightly longer;
- generate a wave induced current;
- show a crest-trench asymmetry increasing with wave height.

Basic idea of the following derivation is the continuity of the information content of the wave group. Also the total energy of the wave group is invariant during propagation despite its local amplitudes, phase relations and total length are changing. For characterizing the propagation of the nonlinear wave group we introduce the new parameter "wave information" $I$ — and assign it to the linear complex water particle surface velocity of a deep water wave (real part = horizontal, imaginary part = vertical).

First the procedure is applied to analyze the propagation of high regular waves, and will then be expanded to nonlinear wave packets. In case of a regular (high) wave the complex wave information $I$ is simply given by the harmonic oscillation

$$I(t,x,w_d,\omega) = w_d \cdot e^{-i(kx-\omega t)}, \quad (11)$$

which corresponds to the orbital surface velocity $w_d = \zeta_d \omega$ of a linear deep water wave. For a regular wave with circular frequency $\omega$, amplitude $\zeta_d$, in water depth $d$ the velocity potential and the surface elevation according to linear wave theory are given by

$$\Phi(t,x,z) = \frac{\zeta_d g \cosh[k(z+d)]}{k} \sinh(kd) \sin(kx-\omega t), \quad (12)$$

$$\zeta(t,x) = \zeta_d \cos(kx-\omega t), \quad (13)$$
g is the acceleration due to gravity, \( t \) denotes time, and \((x, z)\) are space coordinates.

The particle velocities follow from the potential:

\[
\hat{u} = \frac{\partial \phi}{\partial x}, \quad \hat{w} = \frac{\partial \phi}{\partial z}.
\]

(14)

From these velocities the particle elevations are derived using the Lagrangian frame [18]. For the domain \( \Omega \) at time \( t = 0 \) and \((x_0, z_0) \in \Omega \) starting position of each particle, the positions at \( t \geq 0 \) are:

\[
\begin{align*}
t = 0 : & \quad (x, z) = (x_0, z_0), \\
t > 0 : & \quad (x, z) = (x_0 + \alpha(t; x_0, z_0), z_0 + \gamma(t; x_0, z_0)),
\end{align*}
\]

(15)

with \( \alpha \) and \( \gamma \) as particle elevations from rest in horizontal and vertical direction. Then

\[
\frac{d\alpha}{dt} = \frac{\zeta_0 \omega \cosh[k(z_0 + \gamma + d)]}{\sinh(kd)} \cos(k(x_0 + \alpha) - \omega t),
\]

(16)

\[
\frac{d\gamma}{dt} = \frac{\zeta_0 \omega \sinh[k(z_0 + \gamma + d)]}{\sinh(kd)} \sin(k(x_0 + \alpha) - \omega t).
\]

(17)

Expanding the linear wave terms in (16) and (17) at the real position of the respective particle results in the particle displacement of a high wave:

\[
\begin{align*}
\xi(t, x, z) &= \int_0^t \left[ \frac{\tanh[k(\xi(t) + d)]}{\tanh(kd)} \cdot \frac{\cosh[k(z + \xi(t) + d)]}{\sinh(kd)} \cdot w_z \cos[-k(x + \pi(t))] + \omega t] \right. \\
&\quad \left. \cdot \text{Re} I(t, x, w_z, \omega) \right] d\tau.
\end{align*}
\]

(18)

\[
\begin{align*}
\zeta(t, x, z) &= \int_0^t \left[ \frac{\tanh[k(\zeta(t) + d)]}{\tanh(kd)} \cdot \frac{\sinh[k(z + \zeta(t) + d)]}{\sinh(kd)} \cdot w_z \sin[-k(x + \pi(t))] + \omega t] \right. \\
&\quad \left. \cdot \text{Im} I(t, x, w_z, \omega) \right] d\tau.
\end{align*}
\]

(18)

k is the wave number, x and z are starting positions \((\xi(t = 0, x, z) = 0, \zeta(t = 0, x, z) = 0)\) of the water particle before the wave has arrived, and \(x + \xi(t_{end}, x, z), z + \zeta(t_{end}, x, z)\) are the end positions of the particles after the wave has passed. The first terms of Eq. 18 are empirical correcting functions which are necessary due to the wave induced variation of water depth \( d \) (shallow water effects). Note that for deep water waves as well as for very small amplitude waves (linear) in shallow water the empirical functions are 1 and may be neglected without affecting the results in Eq. 18 and 22. For higher regular waves in shallow water the validity of these terms is confirmed by experimental (measured) results ([19]).

As shown in Eq. 18 and in Fig. 3, the particle position is a consequence of the orbital motion and convection. The wave information is related to the position \( x + \pi(t) \), and \( \pi(t) \) is the mean value of the excursion due to its corrected convection:

\[
\pi(t) = \frac{\zeta(t = t_{end}, x, z) - \int_0 ^{t_{end}} \int_0 ^d \xi(t, x, w_z, \omega) \frac{d\tau}{t_{end}}}.
\]

(19)

At the first iteration step \( \pi(t) \) is set to 0. The celerity \( c \) of the regular wave is a consequence of the linear celerity,

\[
c_{linear}(d, k) = \sqrt{\frac{g}{k \tanh kd}},
\]

(20)

and the wave induced convection velocity, i.e.

\[
c(d, k) = c_{linear}(d, k) + \frac{\xi(t = t_{end}, x, z = 0)}{t_{end}}.
\]

(21)

During iteration the entire procedure gets started again at Eq. 11.

If switching from this Lagrangian description of orbital motions of individual particles for regular (high) waves to Eulerian description that gives the motion characteristics at a specified location \( P(x_0, z_0) \) (with different particles passing by) we obtain from Eqs. 11-21

\[
\xi(t, x_0, z_0) = \int_0 ^{t_{end}} \left[ \left( \frac{\tanh[k(\xi(t) + d)]}{\tanh(kd)} \right) \cdot \frac{\cosh[k(z_0 + \xi(t) + d)]}{\sinh(kd)} \right] d\tau.
\]
\[
\frac{\partial^2 w}{\partial t^2} \cos(-kx_0 + \omega(t - \frac{\xi(t)}{c})) \) \, dt;
\]
\[
\tilde{\zeta}(t,x_0, z_0) = \int_0^{\tau=t} \left[ \frac{\tanh[k(\zeta(t) + d)]}{\sinh(kd)} \right] \frac{\zeta(t) + d}{\sinh(kd)} \, dt.
\] (22)

\(\xi(t)\) and \(\zeta(t)\) are mutually dependent. Eq. 22 is iteratively solved in time domain.

So far the application of the "wave information" technique for analyzing the propagation of high regular waves has been illustrated. In order to extend our model to transient wave trains the Fourier transform \(W(x_0, \omega_i)\) of the fictive velocities (derived from linear theory) is selected as the significant quantity which controls the shape of a wave group during propagation.

With the terms \(p_{kj}\) and \(q_{kj}\)

\[
p_{kj} = \frac{\tanh[k \cdot (\zeta + d)]}{\sinh(kd)} \cdot \cos[k \cdot (z + \zeta + d)]
\] (23)

\[
q_{kj} = \frac{\tanh[k \cdot (\zeta + d)]}{\sinh(kd)} \cdot \sinh[k \cdot (z + \zeta + d)]
\]

as a function of time and frequency \((k = 0, 1, 2, \ldots, n - 1\) are time steps and \(j = 0, 1, 2, \ldots, \frac{\pi}{2}\) are frequency steps) we are able to calculate the actual particle velocities

\[
u(t_0 + i \Delta t)_{i=0,\ldots,n-1} = \frac{1}{2\pi} \sum_{k=0}^{n-1} \delta_{ki} \sum_{j=0}^{n-1} W(x_0, j \Delta \omega) \cdot e^{-\frac{j^2 \pi^2}{2}} \cdot p_{kj}
\] (24)

\[
w(t_0 + i \Delta t)_{i=0,\ldots,n-1} = \frac{1}{2\pi} \sum_{k=0}^{n-1} \delta_{ki} \sum_{j=0}^{n-1} W(x_0, j \Delta \omega) \cdot q_{kj}
\] (25)

respectively the particle motions

\[
\xi(t,x,z) = \int_0^{\tau=t} u(\tau, x + \tau) \, d\tau
\]

\[
\zeta(t,x,z) = \int_0^{\tau=t} w(\tau, x + \tau) \, d\tau.
\] (26)

Celerity, i.e. the propagation velocity of information, follows from linear celerity and an additional term due to nonlinear

\[
c_1(t,x) = c_{\text{linear}}(t,x) + c_{\text{add}}(t,x)
\] (27)

\((c_{\text{linear}} = f(\omega(t), k(t)), c_{\text{add}} = \tilde{x}(t,x,z = 0))\)

On this basis, all nonlinear elevations, motions, velocities, and accelerations of nonlinear transient wave trains can be predicted for any time and location. The additional celerity (see Eq. 27) depends on wave frequency and the wave height which follows from the position of the wave group. Even if it is only 5% or less of the value for the linear celerity \(c_{\text{linear}}\), it influences the phase shift significantly.

The transformation of the linear wave information \(I(t,x_0)\) from one position to any other location is similar to the transformation of a linear wave packet: only linear wave celerity has to be replaced by nonlinear "information" speed \(c_1(t,x)\) (see Eq. 27).

It is shown above that the description of nonlinear waves is a result of the calculation of particle tracks using the Lagrangian frame. Finally, surface elevation is asymmetric, with steep crests and flat troughs. In addition, the particle paths are no longer closed as the orbital motion is superimposed on the convective flow in the direction of wave propagation. As an example, Fig. 4 shows the genesis of a high rogue wave using he presented procedure for its generation.

\[\text{Figure 4. GENESIS OF A 3.2 m ROGUE WAVE (WATER DEPTH} \quad \text{d} = \quad \text{4 m).}\]

\[\text{Description by Stokes Third and Higher Order Analogy.} \text{ Similar results follow from the Laplace equation if nonlinear surface boundary conditions are introduced. If wave elevation and velocity potential are expanded as power series,}\]

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with wave steepness $k\zeta_a$ being a small perturbation parameter, we obtain the Stokes high order solutions. In case of a third order Stokes wave in deep water the surface elevation results as follows:

$$\zeta(t) = \frac{1}{k}(k\zeta_a \cos \theta + \frac{1}{2}(k\zeta_a)^2 \cos 2\theta + \frac{3}{8}(k\zeta_a)^3 \cos 3\theta), \quad (28)$$

$$\theta = kx - \omega t.$$  

The linear first term – the Airy wave – is modulated by small higher order terms which steepen the crest and induce a convective flow.

In case of a transient wave packet, the linear term governs propagation. Close to the wave board, we register a long and low wave train. Due to small steepness a linear description is justified, i.e. the surface elevation can be expressed by

$$\zeta(t_i, x_l) = \frac{1}{2\pi} \sum_j F(\omega_j, x_l) e^{i\omega_j t_i} \Delta \omega \quad (29)$$

in discrete form, with $F$ as Fourier transform of the linear wave train (wave information). The amplitude of the harmonic wave is substituted by the envelope $a(t_i)$ of a linear wave packet as in case of a regular wave the amplitude is equivalent to envelope ($a_k(t) = \zeta_a$). The wave packet envelope is calculated by Hilbert transform:

$$a(t_i) = \sqrt{\zeta^2(t_i) + \left(\sum_j F(\omega_j, x_l) e^{i\omega_j t_i - \frac{\pi}{4}}\Delta \omega\right)^2}. \quad (30)$$

Following the Stokes-III analogy, the surface profile is expressed by

$$\zeta(t_i) = \frac{\Delta \omega}{2\pi} \sum_j [F(\omega_j, x_l) e^{i\omega_j t_i + \phi}] \quad (31)$$

$$+ \frac{a(t_i)k(\omega_j)}{2} [F(\omega_j, x_l) e^{2i\omega_j t_i + \phi}]$$

$$+ \frac{3}{8} a^2(t_i)k(\omega_j) [F(\omega_j, x_l) e^{3i\omega_j t_i + \phi}]$$

with $\phi$ as the phase spectrum of the linear wave train (wave information). Figure 5 presents an example of the linear wave elevation as well as the Stokes wave packet. The Stokes wave packet shows significantly steeper wave crests. Comparing associated Fourier spectra we observe that the Stokes wave packet contains higher frequency terms.

Both presented principles for a nonlinear description of waves can be adopted for a nonlinear description of irregular seas.

APPLICATION FOR MODEL TESTS

In order to illustrate the potential of the described wave generation and analysis techniques the interaction of waves and a transparent vertical structure is presented in [19]. Extremely high waves up to 3.2 m of wave height have been generated at the Large Wave Tank (GWK) in Hanover (Fig. 4).

One main advantage is the calculation of a given target wave train at arbitrary positions. Thus, realistic storm scenarios can be simulated such as the New Year Wave shown in Fig. 6 ([21], [22]).

The same transformation procedure is used for capsizing tests at the Hamburg Ship Model Basin (Figs. 7 and 8). In order to simulate rogue wave events (see for example [23] concerning this subject) a high wave group has been integrated into the ITTC seaway by using the mentioned techniques.

CONCLUSIONS

In this paper a new approach for the analysis of nonlinear transient wave trains is proposed. It is based on the definition of a complex wave information which corresponds to linear wave theory. Combined with a modified wave celerity (information speed) the propagation of high wave groups can be calculated using empirical terms based on common wave theories as Stokes.
As a result, nonlinear registrations and associated wave group data are derived from the respective wave information. This procedure allows the evaluation of all relevant wave parameters (motion, pressure distribution, velocities, accelerations), to be used for the analysis of offshore structures.

The presented principles can be extended to natural wave scenarios like the New Year Wave or arbitrary high wave groups within a random sea. Thus, computer controlled capsizing tests with steep design wave trains have been performed. The calculation of the wave train at time dependent positions of the ship allows to relate wave excitation to nonlinear ship responses.

ACKNOWLEDGMENT

The development of the nonlinear wave generation technique has been funded by the Federal Ministry of Research and Education - BMBF - for several years. The most recent related project dealt with analytical and numerical methods for generating and optimizing "tailor-made" wave trains and its application to capsizing tests. The authors wish to thank the sponsor for the long-standing support which allowed us to develop and improve our technique and expand it to include nonlinear effects of very high wave groups.

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Figure 8. ROLL MOTION OF A RO-RO VESSEL IN A SEVERE STORM 
WAVE TRAIN WITH EMBEDDED HIGH TRANSIENT WAVE ($T_p = 2.5$ s, $H_s = 0.45$ m) at $GM_{mod} = 0.040$ m, $v_{mod} = 1.3$ m/s. ABOVE: WAVE 
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