NONLINEAR DYNAMICS OF TOWED UNDERWATER VEHICLES
– NUMERICAL MODELLING AND EXPERIMENTAL VALIDATION –

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ABSTRACT

Controllable towed underwater vehicles are used as carrier systems in a wide range of maritime applications. The quality and accuracy of the registered multimedia seabed and subsoil data depends on the dynamic behaviour and stability of the towed sensor carrier as well as on the transmission of wave induced motions along the submarine umbilical. This paper presents a comprehensive numerical and experimental analysis of the dynamic behaviour of towed systems, taking into account the non-linear characteristics of drag and lift coefficients as well as the non-linear dynamic cable force. It is proved that ship motions are inducing substantial cable oscillations resulting in significant excitation of the towed body. A computer program is developed to determine the cable curvature for stationary conditions as well as the dynamic response of the towed body due to ship oscillations. The numerical model is validated by towing tests, performed at the TU Berlin in one of the world largest cavitation and circulation tunnels with free surface.

KEYWORDS

Towed underwater vehicle, Nonlinear dynamics, Nonlinear cable forces, Orbital oscillations, Sensor carrier, Depressor wings, Numerical simulation, Experimental validation

INDRODUCTION

Vertically tethered systems or towed bodies – applied in a wide spectrum of offshore operations – are characterised by hydroelastic effects. In general, such systems consist of the components research or support vessel, tether, umbilical or towing cable, and a suspended load or a towed sensor carrier (Figure 1). Tethers connecting the ship with the subsea unit allow the precise deployment of the suspended equipment like drill strings, subsea manifolds, BOP stacks or templates at a pre-selected location on the seabed. Umbilicals

Figure 1: Typical components of a towing system
enable a ROV or towed body to stay for a long time at the underwater work site and allows real-time communication.

The quality and suitability of the recorded multimedia seabed and subsoil data for further analysis depends on the dynamic behaviour and stability of the towed sensor carrier as well as on the transmission of wave induced motions along the submarine umbilical. The tether or umbilical couples the motion of the floating structure with the oscillations of the underwater system. Under certain conditions resonance is observed, with substantially increased motion and acceleration amplitudes. Due to high inertia and drag forces the deep dived module cannot follow the umbilical oscillations, with the consequence of cable slackening (Clauss and Hoog (2001)). In this case the cable tension decreases to zero, and the motions of the ship and the submerged structure are decoupled for a short while. During this time the upper cable end and the towed body are oscillating independently with different amplitude and phase. When the cable becomes taut again, large snap loads and erratic motions of the submerged body occur resulting in severe damage of the umbilical with its internal electrical and optical conductors. The resulting degradation reduces the life span of the umbilical and endangers the recovery of the suspended or towed subsea module.

A validated analysis model is required, based on theoretical investigations of the non-linear cable dynamics, using the method of finite elements. A dynamic time-domain simulation of marine operations of vertically tethered or towed systems allows the evaluation of occurring load and motion amplitudes during specific missions in advance, defining operational limitations due to weather conditions giving rise to snap loads. Furthermore, designers are enabled to minimise motions and to limit tensions by choosing optimal system parameters.

The dynamic analysis of deep dived towed bodies comprises the determination of the dynamic behaviour of vertically suspended loads. In addition, it considers the hydrodynamic forces and moments due to stationary towing speed as well as hydroelastic effects. Ablow and Schechter (1983), and Milinazzo et al. (1987) use the lumped-mass-method to analyse the three-dimensional dynamic behaviour of submerged towed bodies and to determine the trajectory during a given ship manoeuvre.

Wu and Chwang (1997) describe the three-dimensional dynamic behaviour of a towing system subdivided into a primary and a secondary cable. They show that the heave and pitch motions of the sensor carrier attached to the secondary cable is significantly damped compared to the oscillations of the depressor body at the lower end of the primary cable. Buckham et al. (1999) develop a numerical model – based on the lumped-mass-method – to calculate the three-dimensional dynamic behaviour, i.e. the velocity and depth profile as well as the horizontal trajectory of a towed body during a 180°-manoeuvre, considering the influence of active control surfaces at the tail plane.

Clauss et al. (1998) determine experimentally and numerically the seaway dependent limitations of offshore pipeline operations. The applied numerical formulation – based on the finite element method – is split in a simplified stationary approach by solving only the stationary components of the differential equations. In a second step the oscillation around the stationary equilibrium – caused by forced excitation at the upper pipeline end – is determined.

This paper presents a comprehensive numerical and experimental analysis of the dynamic behaviour of towed systems, based on the numerical model for pipelaying analysis of Claus et al. (1998), taking into account the non-linear characteristics of drag and lift coefficients of the sensor carrier as well as non-linear components of dynamic forces due to cable curvature. The analysis proves that ship motions are substantially transmitted along the submerged cable, and excite oscillations of the towed body. A numerical model is developed to determine both, the cable curvature for stationary conditions as well as the dynamic response of the towed body due to ship oscillations. The numerical model is validated by towing tests, performed at the TU Berlin in one of the world largest cavitation and circulation tunnel with free surface. The motions of the towed model are recorded by video cameras and evaluated by digital video post-processing. Cable forces are measured at the upper suspension point (winch) and at the lower end (body attachment).
NUMERICAL ANALYSIS

Coordinate Systems

For numerical analysis the cable is subdivided in small segments of length $ds$. The motion of each cable segment is described by the vector $r(s,t)$, depending on the cable coordinate $s$ and time $t$. The cable curve can be split in a stationary and a dynamic component, i.e.

$$r(s,t) = r_s(s) + u(s,t).$$

(1)

Two coordinate systems (Figure 2) are introduced: The first system with the components $X,Y,Z$ travels with constant towing speed $U$. $X$ points forward in towing direction, $Y$ points to port and $Z$ points vertically upwards. The second, body fixed coordinate system is located at the respective cable position $s$, distorted against the first coordinate system by the inclination angle $\phi(s)$ and the deviation angle $\psi(s)$. The transformation is performed, using the transformation matrix $A$ as follows:

$$
\begin{bmatrix}
\xi_S(s) \\
\xi_B(s) \\
\xi_N(s)
\end{bmatrix} =
A
\begin{bmatrix}
\xi_X \\
\xi_Y \\
\xi_Z
\end{bmatrix},
$$

where

$$A =
\begin{bmatrix}
\cos \phi(s) \cos \psi(s) & \cos \phi(s) \sin \psi(s) & \sin \phi(s) \\
-\sin \psi(s) & \cos \psi(s) & 0 \\
-\sin \phi(s) \cos \psi(s) & -\sin \phi(s) \sin \psi(s) & \cos \phi(s)
\end{bmatrix}.
$$

(2)

The local derivatives are therefore:

$$
\xi'_S(s) = \phi' \cdot \xi_N + \psi' \cos \phi \cdot \xi_B; \quad \xi'_B(s) = -\phi' \cdot \xi_S - \psi' \sin \phi \cdot \xi_B; \quad \xi'_N(s) = \psi' (\sin \phi \cdot \xi_N - \cos \phi \cdot \xi_S).
$$

(3)

Hydrodynamic cable load

The line load due to viscous drag $q_D$ is a superposition of the tangential and normal drag force which are evaluated with empirical tangential and normal drag coefficients $c_D$ and $c_N$, respectively, acting at each finite cable element in the plane opened up by the cable tangent $\xi_S$ and the relative free flow direction $v_{rel}$:

$$
\begin{bmatrix}
q_{DS} \\
q_{DB} \\
q_{DN}
\end{bmatrix} = \rho D 2 \begin{bmatrix}
\pi \cdot c_S \cdot v_{rel,S} & 0 & 0 \\
0 & c_D \sqrt{V_{rel,B}^2 + V_{rel,N}^2} & 0 \\
0 & 0 & c_D \sqrt{V_{rel,B}^2 + V_{rel,N}^2}
\end{bmatrix} \begin{bmatrix}
v_{rel,S} \\
v_{rel,B} \\
v_{rel,N}
\end{bmatrix}.
$$

(4)

The relative free flow velocity $v_{rel}$ follows from two components, the steady towing speed $U$ and the cable oscillation $\xi$:

$$v_{rel,S} = -U \xi_X - \xi; \quad v_{rel,B} = U \sin \psi - \xi_B; \quad v_{rel,N} = U \sin \phi \cos \psi - \xi_N.
$$

(5)

In addition, the ribbed surface of armored umbilicals causes a transverse force $q_{AL}$ normal to the relative free flow direction $v_{rel}$. This asymmetrical load is taken into account by introducing the empirical transverse force coefficient $c_{AL}$, i.e.

$$
\begin{bmatrix}
q_{ALS} \\
q_{ALB} \\
q_{ALN}
\end{bmatrix} = c_{AL} \rho D 2 v_{rel,S} \begin{bmatrix}
0 \\
v_{rel,N} \\
v_{rel,B}
\end{bmatrix} = c_{AL} \rho D 2 v_{rel,S} \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & -1 & 0
\end{bmatrix} \begin{bmatrix}
v_{rel,S} \\
v_{rel,B} \\
v_{rel,N}
\end{bmatrix}.
$$

The inertia force $q_I$ is derived from the general Morison-equation (with $c_A = 1$):
\[
q_{\tau} = -c_{A} \rho \frac{\pi D^{2}}{4} \left[ 0 \begin{array}{c}
0 \\
1 \\
0 
\end{array} \right]
\begin{bmatrix}
\hat{u}_{S} \\
\hat{u}_{N} 
\end{bmatrix} = -c_{A} \rho \frac{\pi D^{2}}{4} \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 1 \\
0 & 0 & 1 
\end{bmatrix} \begin{bmatrix}
\hat{u}_{S} \\
\hat{u}_{N} 
\end{bmatrix}.
\]

In conclusion, the hydrodynamic load follows from the sum of all components:
\[
q_{0} = q_{\tau} + q_{D} + q_{A} = -m_{hy} \cdot \dot{\hat{v}} + \hat{b} \cdot (-U \hat{e}_{X} - \hat{r}),
\]
where \(m_{hy}\) denotes the hydrodynamic mass tensor and \(\hat{b}\) the hydrodynamic drag and lift tensor, i.e.
\[
[m]_{hy} = c_{A} \rho \frac{\pi D^{2}}{4} \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 1 \\
0 & 0 & 1 
\end{bmatrix} ; [\hat{b}] = \frac{\rho D}{2} \begin{bmatrix}
\pi \cdot c_{S} |v_{rel,S}| & 0 & 0 \\
0 & c_{D} \sqrt{v_{rel,B}^{2} + v_{rel,N}^{2}} & c_{A} \cdot c_{rel,S} \\
0 & -c_{A} \cdot c_{rel,S} & c_{D} \sqrt{v_{rel,B}^{2} + v_{rel,N}^{2}} 
\end{bmatrix}.
\]

The velocities \(\hat{u}\) as well as the accelerations \(\hat{\ddot{u}}\) of the incremental cable displacements vanish for stationary conditions, leading to the relative stationary free flow velocity of \(v_{rel} = -U \hat{e}_{X} \cdot \hat{e}_{s}\), \(-U \hat{e}_{X} \cdot \hat{e}_{B}\), \(-U \hat{e}_{X} \cdot \hat{e}_{N}\) (see Eqn. (4)). The corresponding hydrodynamic load follows from Eqn. (5), and for stationary conditions – results in:
\[
\begin{bmatrix}
q_{st} \\
q_{st} \\
q_{st} 
\end{bmatrix} = \frac{\rho DU}{2} \begin{bmatrix}
-\pi \cdot c_{S} \cos^{2} \varphi \cos^{2} \psi \\
- c_{D} \sqrt{1 - \cos^{2} \varphi \cos^{2} \psi} \sin \psi - c_{A} \cos \varphi \sin \varphi \cos^{2} \psi \\
- c_{D} \sqrt{1 - \cos^{2} \varphi \cos^{2} \psi} \sin \varphi \cos \psi + c_{A} \cos \varphi \cos \psi \sin \psi 
\end{bmatrix}.
\]

\textbf{Cable elongation}

Cable elongation \(\varepsilon\) comprises a stationary and a non-steady component: \(\varepsilon = \varepsilon_{st} + \varepsilon_{dy}\), with:
\(\varepsilon_{st} = F_{st} \cdot EA\). At stationary conditions the finite cable element length is \(|dr_{st}| = ds\), at non-steady conditions it is elongated to \(|dr_{st}| + du\) = \(|r'_{st} + u'|ds = \varepsilon_{s} + u'|ds\) (with: \(r'_{st} = \varepsilon_{s}\)).
\[
\varepsilon_{dy} = \frac{|dr| - |dr_{st}|}{|dr_{st}|} = \frac{|\varepsilon_{s} + u'|ds - ds}{ds} = \varepsilon_{s} + u' - 1 = \sqrt{\varepsilon_{s}^{2} + 2 \varepsilon_{s} u' + u'^{2}} - 1 = \frac{1}{2} u'^{2}.
\]

\textbf{Cable equation}

By definition, bending and torsion moments are not transferred via an ideal cable: Therefore the remaining force at the free margins of a cable element is the axial force only. Thus the equation of motion for a cable element of submerged weight \(w\) is:
\[
m_{col} \ddot{\hat{r}} - (F'_{r})' = w + q_{0}.
\]
Introducing the hydrodynamic load vector \(q_{0}\) from Eqn. (5) results in:
\[
m_{col} \ddot{\hat{r}} + \hat{b} \cdot \dot{\hat{r}} - (F'_{r})' = w + q,
\]
with: \(m = m_{col}(E) + m_{hy}\) and the stationary line load \(q = q_{st} + q_{dy}\).

\textbf{Evaluation of the stationary cable curve}

Considering the towing system at constant towing speed, excluding any external excitation at the upper cable end, the non-steady terms \(\dot{\hat{r}}, \ddot{\hat{r}}\) vanish. With \(q = q_{st}\) Eqn. (7) reduces to
\[-(F_{st} r'_{st})' = -w e_{s} + q_{st},\]
leaving only the stationary components of the line load and the axial cable force. With \(r'_{st} = e_{s}\) and Eqn. (3) the above equation is transformed into the body fixed coordinate system:
\[ -(F_u \varphi_S)' = -w(\sin \varphi_S + \cos \varphi_S \varphi_N) + q_{st} \]
\[ \Rightarrow -F_u' \varphi_S = -F_u (\varphi_N' + \varphi' \cos \varphi_E) = -w(\sin \varphi_S + \cos \varphi_S \varphi_N') + q_{st}. \]

Splitting Eqn. (8) into the three directional components $S, B, N$ and introducing the stationary hydrodynamic load from Eqn. (6), the characteristics of the stationary cable curve are derived from:

\[
F_u'(s) = w \sin \varphi(s) + \frac{1}{2} \pi \rho S DU^2 \cos^2 \varphi \cos^2 \psi(s),
\]

\[
\varphi'(s) = \frac{w \cos \varphi(s) - \frac{1}{2} \rho DU^2 (c_D \sin \varphi \cos \psi \sqrt{1 - \cos^2 \varphi \cos^2 \psi} + c_{st} \cos \varphi \cos \varphi \sin \psi)}{F_u(s)},
\]

\[
\psi'(s) = \frac{\frac{1}{2} \rho DU^2 (-c_D \sin \psi \sqrt{1 - \cos^2 \varphi \cos^2 \psi} + c_{st} \cos \varphi \sin \psi \cos^2 \varphi)}{F_u(s) \cos \varphi(s)}.
\]

Integration starts at the towed body suspension point, where the axial cable tension is equivalent to the sum of body weight, drag and hydrodynamic depressor force. Consequently the direction of the resulting force vector corresponds to the inclination and deviation angle of the first finite element. Since the integration starts at the towed end, all cable nodes have to be shifted down by the vertical stationary displacement of the last integrated node point, in order to adjust the winch point at water surface level.

**Solution for non-steady conditions**

To determine the displacements of each cable node – caused by external excitations at the upper cable end – only the non-steady components of Eqn. (7) are considered:

\[
m \cdot \ddot{u} + b \cdot \dot{u} - F \cdot \dot{u} = q_{dy}
\]

with

\[
F_{dy} = F_{st} u' + F_{dy} (\varphi_N' + u')
\]
\[
q_{dy} = -U(b - b_{st}) \cdot e_X.
\]

Applying the Galerkin-procedure, the equation for each cable segment is multiplied with a kinematical possible displacement $\delta u'$ and integrated over the whole cable length:

\[
\int_L \left( \delta u \cdot m \cdot \ddot{u} + \delta u \cdot b \cdot \dot{u} - \delta u \cdot F \cdot \dot{u} \right) ds = \int_L \delta u \cdot q_{dy} ds.
\]

Partial integration results in:

\[
\int_L \left( \delta u \cdot m \cdot \ddot{u} + \delta u \cdot b \cdot \dot{u} - \delta u \cdot F \cdot \dot{u} \right) ds = \int_L \delta u \cdot q_{dy} ds + \left[ \delta u \cdot F_{dy} \right]_0^L.
\]

Non-steady cable force oscillations are expressed by displacement quantities, according to the law of elasticity:

\[
F_{dy} = (F_{st} + F_{dy})(\varphi_N' + u') - F_{st} \varphi_N' = F_{st} (\varphi_S + u') \quad \text{where} \quad F_{dy} = E(A' \cdot \varphi_S + u'^2 / 2).
\]

Note, that the dynamic cable force is split into a linear and a non-linear component:

\[
F_{dy} = F \cdot u' + F_{nl}
\]

with:

\[
F = ((F_{st} + F_{dy})(\varphi_N' + u') + E A_{dy} \varphi_S \otimes \varphi_S), \quad F_{dy} = E(A' \cdot \varphi_S + u'^2 / 2); \quad F_{nl} = E A \cdot u'^2 / 2 \cdot \varphi_S,
\]

transforming Eqn. (9) into

\[
\int_L \left( \delta u \cdot m \cdot \ddot{u} + \delta u \cdot b \cdot \dot{u} + \delta u \cdot F_{nl} \right) ds = \int_L \left( \delta u \cdot q_{dy} - \delta u \cdot F_{dy} \right) ds + \left[ \delta u \cdot F_{dy} \right]_0^L.
\]
The sum of the virtual displacements with respect to tangential and normal components of the stationary cable curve are

\[
\int \delta \{u\}^T \{m\} \{\ddot{u}\} + \delta \{u\}^T \{b\} \{\ddot{u}\} + \delta \{u\}^T \{F\} \{\dot{u}\} \cdot ds = \int \delta \{u\}^T \{q\} dy - \delta \{u\}^T \{F_{\text{el}}\} \cdot ds + \delta u \cdot F_{\text{phi}} \frac{L}{b},
\]

(12)

\{u\}^T = \begin{bmatrix} u_S & u_B & u_N \end{bmatrix}

denotes the components of the non-steady displacement, and its derivative is:

\[
\begin{bmatrix} u' \cr u_B' \cr u_N' \end{bmatrix} = \begin{bmatrix} \varphi' \sin \varphi & -\varphi' \cdot \cos \varphi \end{bmatrix} \begin{bmatrix} u_S \cr u_B \cr u_N \end{bmatrix} = \{u\}' + [\kappa] \cdot \{u\}

\text{with } [\kappa] = \begin{bmatrix} 0 & d_N & -d_B \\ -d_N & 0 & d_S \\ d_B & -d_S & 0 \end{bmatrix}.
\]

From Eqn. (10) we obtain the forces

\[
[F] = \begin{bmatrix} EA + F_{st} + F_{dy} & 0 & 0 \\ 0 & F_{st} + F_{dy} & 0 \\ 0 & 0 & F_{st} + F_{dy} \end{bmatrix} \quad \text{and} \quad \{F\}_{el} = \frac{1}{2} EA \{u\}' \{u\}',
\]

\[\{m\}, \{b\}, \{q\}_{\text{phi}} \text{ in Eqn. (12) are expressed as }\]

\[
\begin{bmatrix} m \end{bmatrix} = m_{\text{cab}} \begin{bmatrix} m \end{bmatrix}_{by} + \begin{bmatrix} m \end{bmatrix}_{by} = \rho D \frac{\alpha D^2}{4} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \begin{bmatrix} b \end{bmatrix} = \rho D \frac{\alpha D^2}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -c_{\text{rel,normal}} & c_{\text{rel,normal}} & c_{\text{rel,normal}} \end{bmatrix}
\]

with:

\[
\begin{bmatrix} v_{\text{rel,normal}} \\ v_{\text{rel,normal}} \end{bmatrix} = \sqrt{(U \sin \varphi - u_B)^2 + (U \sin \varphi \cos \varphi - u_N)^2}; \quad \begin{bmatrix} q \end{bmatrix}_{\text{phi}} = -U \begin{bmatrix} [b] - [b] \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
\]

Since this formulation is valid for the entire cable, it will be applied to an individual finite element at the position \( s_i < s < s_{i+1} \). In this case the degrees of freedom are just the translatory motions at each end. Between the two end points (nodes) the displacement is interpolated linearly. A normalized coordinate \( \xi \) is introduced to integrate along the finite element. With consideration to the paper limitations the explicit integration formulation over the finite element cannot be explained in detail.

**Towed body**

The dynamic behaviour of the towed body is described by rigid body equations of motion, corresponding to a conventional airplane configuration, i.e. all translatory and rotatory degrees of freedom are considered. The motions of the first cable node are identical with the translatory motion of the rigid body. Due to the additional rotatory motion of the towed body the number of system degrees of freedom increases by three, and the equation of motion of the rigid body results in:

\[
\frac{m_{\text{bDY}}}{m_{\text{bDY}}} \ddot{u}_{\text{bDY}} + \frac{\overline{b}_{\text{bDY}}}{m_{\text{bDY}}} \dddot{u}_{\text{bDY}} + \frac{c_{\text{bDY}}}{m_{\text{bDY}}} u_{\text{bDY}} = \sum \begin{bmatrix} F_a \\ M_a \end{bmatrix},
\]

(13)

The external force \( F_a \) is equivalent to the cable force at the towed body suspension point. The displacement vector \( u_{\text{bDY}} \) relates to the coordinate system XYZ:

\[
\{u\}_{\text{bDY}} = \begin{bmatrix} u_X & u_Y & u_Z & \phi & \theta & \gamma \end{bmatrix}.
\]

The mass-matrix considers that the towed body is symmetric along the XZ-plane. Horizontal and vertical distances between the cable suspension point and centre gravity are \( x_g \) and \( z_g \).
\[
\begin{bmatrix}
    m + m_{a11} & 0 & 0 & 0 & m \cdot z_g & 0 \\
    0 & m + m_{a22} & 0 & -m \cdot z_g & 0 & m \cdot x_g \\
    0 & 0 & m + m_{a33} & \Theta_{zz} + m \cdot z_g^2 + m_{a44} & 0 & -m \cdot x_g - m_{a55} \\
    0 & -m \cdot z_g & 0 & 0 & \Theta_{yy} + m(x_g^2 + z_g^2) & 0 \\
    m \cdot z_g & 0 & -m \cdot x_g - m_{a53} & 0 & +m_{a55} & \Theta_{xx} - m(x_g \cdot z_g) \\
    0 & m \cdot x_g & 0 & -\Theta_{xy} - m(x_g \cdot z_g) & 0 & \Theta_{xx} + m \cdot x_g^2 + m_{a66}
\end{bmatrix}
\]

The damping matrix of the towed body has the following hydrodynamic drag and lift components. Note the non-linear drag and the additional lift force with larger angles of incidence.

\[
\begin{bmatrix}
    -\left( C_{dx} + \frac{\partial C_{dx}}{\partial \alpha} \theta^2 \right) \cdot |u_X| \\
    0 & 0 & 0 & 0 & 0 & 0 \\
    0 & -C_{dy} \cdot |u_Y| & 0 & 0 & 0 & 0 \\
    \left( C_{L_0} + \frac{\partial C_{L_0}}{\partial \alpha} (\theta - \alpha') \right) \cdot |u_{rel}| & 0 & -C_{dz} \cdot |u_Z| & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & -C_{do} \cdot |l_\theta| & 0 \\
    0 & 0 & 0 & 0 & 0 & -C_{dy} \cdot |\hat{\psi}| \cdot l_Y
\end{bmatrix}
\]

with: \( \alpha' = \arctan \left( \frac{\hat{u}_Z}{\hat{u}_X} \right) \).

The stiffness-matrix of the towed body considers the restoring moments due to the stabilising tailplane around the pitch and yaw axes:

\[
c_{b_{hy,55}} = c_{b_{hy,56}} = \frac{\rho}{2} A_{b_{hy}} \frac{\partial C_M}{\partial \alpha} \cdot |v_{rel}|^2.
\]

**EXPERIMENTAL VALIDATION**

Model towing tests are performed at the TU Berlin in one of the world largest cavitation and circulation tunnels to validate the non-linear numerical model (Figure 3). The stationary cable curve as well as the orbital oscillating motion of the towed body due to external disturbances at the upper cable end are determined using video postprocessing. Underwater video cameras are attached to a transverse adjustable camera mounting device. The X and Z coordinates of the lenses are known with respect to the water surface and the horizontal winch position. The experimental procedure commences by recording a reference frame to calibrate the dimensions of the obtained video picture in the plane of the submerged towing system. After a constant flow velocity is reached, external excitations are introduced at the upper cable end using a linear oscillator device. Resulting oscillations of the model sensor carrier are recorded. Finally, the recorded reference coordinate system is transformed into a transparent digital bitmap-file and superimposed with the digitised video frames of the towed system. In addition to motion measurements, cable forces at the towed body suspension point and at the upper winch point as well as the horizontal oscillation amplitudes are registered.
RESULTS

Figure 4 and 5 presents stationary cable curves as well as towed body motions and forces comparing experimental and numerical results. Comparison between measured and calculated orbital motion of the towed body illustrate good agreement. Note, that shape variations of the orbital trajectory due to increased excitation frequency and towing speed is observed. The numerical simulations illustrate that non-linear effects are dominating the motion behaviour if the sensor carrier is equipped with depressor wings. In this case the towed body motion becomes substantially flatter, and the associated cable force exhibits high peaks – three times as high as without depressor wings. Furthermore, the dynamic analysis proves that ship motions induce substantial cable oscillations resulting in significant excitations of the towed body with a transfer factor of about one.

CONCLUSIONS

The paper introduces a numerical model to analyse the dynamic response of towed submarine sensor carriers due to wave-induced oscillations at the winch of the towing ship. The evaluation proves that ship motions are substantially transmitted along the towing cable or umbilical resulting in significant excitations of the towed body. Experimental validation of the calculated stationary cable curves, orbital oscillations of the towed body around its stationary origin as well as axial forces at the upper and lower cable ends show good agreement between measured and calculated data. Even super-harmonic cable force oscillations at the towed body suspension point of twice the frequency of the external excitation are determined. It can be stated, that the tense configuration of the stationary cable curve acts like a virtual system-stiffness, since the non-linear cable force component considers the incremental displacement of each finite element as well as cable curvature. This significant non-linear effect has never been considered before.

The dynamic analysis reveals that the shape of the orbital trajectory of the towed body caused by wave-induced excitations depends on the geometric and therefore hydrodynamic configuration of the submarine body. The numerical model which considers the complete rigid body equations of motions accurately predicts the effect of depressor wings considering the non-linear increase of drag force due to a higher angle of incidence or velocity. As a consequence a significant increase in axial cable force is observed.

REFERENCES


Figure 4: Numerical analysis and model test results – stationary cable curve and dynamic behaviour of towed body without depressor wings:

A) $U_{low} = 1.7 \text{ m/s and } T = 2.59 \text{ s}$;  
B) $U_{low} = 2.8 \text{ m/s and } T = 2.99 \text{ s}$
Figure 5: Numerical analysis and model test results – stationary cable curve and dynamic behaviour of towed body with depressor wings:

A) $U_{low} = 1.8 \text{ m/s and } T = 3.87 \text{ s}$;

B) $U_{low} = 2.6 \text{ m/s and } T = 2.34 \text{ s}$