Optimization of Transient Design Waves in Random Sea

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ABSTRACT

Transient design waves in random sea are optimized by a Sequential Quadratic Programming method. The procedure is based on the linear wave superposition model which allows an exact representation of the wave train in frequency domain. For a given design variance spectrum, the desired characteristics of the design wave regime regarding wave height and crest structure are generated by optimizing an initially random phase spectrum. The solution depends on the initial phase values resulting in different wave regimes for the same problem definition. The analysis of the linear wave evolution is expanded to a fully nonlinear simulation using the finite element method based on potential flow theory. Results are shown for a high transient design wave within a tailored group of three successive waves embedded in random sea which is described by the infinite depth TMA spectrum. The maximum wave height of the transient design wave is twice the significant wave height.

KEY WORDS

Transient design waves, random sea, nonlinear waves, freak waves, optimization, numerical wave tank, finite element method

INTRODUCTION

Reliable data on the extreme wave environment is an essential prerequisite for designing safe and economic offshore structures. Air gap, green water, slamming impacts and dynamic instability of ships highlight some engineering problems related to high wave crests. TLP-ringing is believed to be caused by steep transient waves resulting in high stresses and other undesirable effects. Extremely high single waves or wave groups will typically arise within an irregular sea state and are of transient nature.

Deterministic design wave regimes for model tests or numerical simulations need to be described by global parameters as the variance spectrum which is defined by significant wave height and peak period. In addition, local characteristics of the wave pattern and extreme waves are of great interest. The generation of these complex design wave regimes is highly complicated due to the nonlinearity of the free surface.

Stansberg (1990) empirically investigates extreme waves in laboratory generated irregular wave trains. Wave superposition based on linear wave theory is used to simulate randomly occurring large waves which are picked out for experimental study. Baldock and Swan (1994) present a description of a two-dimensional irregular sea state in which a large transient wave is generated by focusing component waves. The numerical method is based upon a Fourier series expansion in space and time which is validated by laboratory data. Taylor et al. (1995) describe a theory to constrain a random time series for generating a large crest elevation of given size at a prescribed time. The technique is a linear process with the extreme surface elevation being indistinguishable from a purely random occurrence of that particular crest. Read and Sobey (1987) point out that the phase spectrum is routinely ignored on the assumption that the Gaussian random wave model is a sufficiently complete description of a field record. Procedures are developed for unwrapping and detrending the phase spectrum to provide insight into the potential identification of ordered structures, possibly wave groups, in the phase spectrum. Wolfram et al. (1994) examine the time series of three severe storms recorded at the Alwyn platform in the northern North Sea. The Fourier analysis of these series shows that the wave components do not have uniformly distributed random phase which may be attributable to bound waves.

In this paper, a high transient design wave within a tailored group of three successive waves in random sea is optimized by Sequential Quadratic Programming applying linear wave theory. The design variance spectrum is chosen to be the TMA spectrum and only the initially random phase spectrum is changed during the optimization. The target wave characteristics are defined by an objective function and constraints which take the maximum stroke, velocity and acceleration of the wave board into account. The velocity of the wave board serves as Neumann boundary condition in the fully nonlinear simulation of the wave evolution in time domain. The numerical procedure is based on potential flow theory and applies the finite element method developed by Wu and Eatock Taylor (1994, 1995).
TIME DOMAIN AND FREQUENCY DOMAIN

A continuous real-valued wave record $\zeta(t)$ may be represented in frequency domain by its complex Fourier transform $F(\omega)$ which is calculated by Eq. (1). Applying the inverse Fourier transformation, Eq. (2), gives the original record $\zeta(t)$:

$$F(\omega) = \int_{-\infty}^{+\infty} \zeta(t) e^{-i\omega t} dt$$  \hspace{1cm} (1)

$$\zeta(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega$$  \hspace{1cm} (2)

where $t$ represents the time and $\omega = 2\pi f$ the angular frequency. In polar notation, the complex Fourier transform can be expressed by its amplitude and phase spectrum:

$$F(\omega) = |F(\omega)| e^{i \arg F(\omega)}$$  \hspace{1cm} (3)

where $i = \sqrt{-1}$ is the imaginary unit. In practice, it is necessary to adopt a discrete and finite form of the Fourier transform pair described by Eqs. (1) and (2):

$$F(r\Delta \omega) = \Delta t \sum_{k=0}^{N-1} \zeta(k\Delta t) e^{-i2\pi r k/N}$$  \hspace{1cm} (4)

$$\zeta(k\Delta t) = \frac{\Delta \omega}{2\pi} \sum_{r=0}^{N/2} F(r\Delta \omega) e^{i2\pi r k/N}$$  \hspace{1cm} (5)

where the values $\zeta(k\Delta t)$ represent the available data points of the discrete finite wave record with $\Delta t$ denoting the sampling rate and $\Delta \omega = 2\pi/(N\Delta t)$ the frequency resolution. The summation in Eqs. (4) and (5) can be efficiently computed by the fast Fourier transform (FFT) and its inverse algorithm (IFFT) (Brigham (1974); Conte and De Boor (1980)).

The Fourier transform $F(\omega)$ and the variance spectrum $E(\omega)$ are related through Parseval’s theorem which states for the single-sided discrete Fourier transform pair

$$E(r\Delta \omega) = \frac{1}{\pi N \Delta t} |F(r\Delta \omega)|^2$$  \hspace{1cm} (6)

The design variance spectrum is chosen to be the finite depth variant of the Jonsswap spectrum known as TMA spectrum (Hassellmann et al. (1973), Bouws et al. (1985)):

$$\frac{\omega^4 E(q)}{g^2} = \alpha q^{-5} \frac{\tanh^2(kd)}{1 + 2kd / \sinh(2kd)} e^{-1.28q^{-4} \gamma^{1.5} q^{-1.7}}$$  \hspace{1cm} (7)

where $q = \omega / \omega_p = f/f_p$, represents the normalized frequency with respect to the peak frequency $f_p = 1/T_p$. The Jonsswap peak enhancement factor $\gamma$ is set to 3.3 and the spectral width parameter $\sigma$, to 0.07 for $q \leq 1$ and 0.09 for $q > 1$ with $r = (q - 1)/\sigma$. The frequency-dependent wave number $k$ is calculated from the dispersion relationship $\omega^2 = g k \tanh(kd)$ where $g$ is the acceleration due to gravity and $d$ the water depth. Since the zeroth spectral moment

$$m_0 = \int_0^{\infty} E(\omega) d\omega = \int_0^{\infty} E(q) dq$$

$$= \alpha g^2 \omega_p^4 \int_0^{\infty} q^{-5} \frac{\tanh^2(kd)}{1 + 2kd / \sinh(2kd)} e^{-1.28q^{-4} \gamma^{1.5} q^{-1.7}} dq$$

$$= \alpha g^2 \omega_p^4 I_0 \left( \gamma, \frac{\omega_d^2}{g} \right)$$ \hspace{1cm} (8)

and the significant wave height are related by $H_s = 4\sqrt{m_0}$, the spectral parameter $\alpha$ is determined from

$$\alpha = \frac{H_s^2 \omega_p^4}{16 g^2 I_0 \left( \gamma, \frac{\omega_d^2}{g} \right)}$$  \hspace{1cm} (9)

OPTIMIZATION

As long as linear wave theory is applied, the sea state can be regarded as superposition of independent harmonic waves, each having a particular direction, amplitude, frequency and phase. For a given design variance spectrum of an unidirectional wave train, the phase spectrum is responsible for all local characteristics, e.g. the wave height and period distribution as well as the location of the highest wave crest in time and space. For this reason, an initially random phase spectrum $\arg F(\omega)$ is optimized to generate the desired design wave train with specified local properties. The phase values $\beta = (\beta_1, \beta_2, \ldots, \beta_n)^T$ are bounded by $-\pi \leq \beta \leq \pi$ and are initially determined from $\beta_j = 2\pi(R_j - 0.5)$ where $R_j$ are random numbers in the interval $0$ to $1$.

The set up of the optimization problem is illustrated for a high transient design wave within a tailored group of three successive waves in random sea. The crest front steepness of the design wave in time domain $\epsilon_t$ as defined by Kjeldsen (1990):

$$\epsilon_t = \frac{2\pi \zeta_{crest}}{g T_{rise} T_{zd}}$$  \hspace{1cm} (10)

is maximized during the optimization process. $\zeta_{crest}$ denotes the crest height, $T_{rise}$ the time between the zero-upcrossing and crest elevation, and $T_{zd}$ the zero-downcrossing period which includes the design wave.

The target zero-upcrossing wave heights of the leading, the design and the trailing wave are defined by $H_l$, $H_d$ and $H_t$. The target locations in space and time of the design wave crest height $\zeta_d$ are $x_{target}$ and $t_{target}$. These data define equality constraints. The maximum values of stroke $\delta_{max}$, velocity $u_{max}$, and acceleration $a_{max}$ of the wave board motion $x(t)$ define inequality constraints to be taken into account. Hence the optimization problem is stated as

$$\text{minimize } f(\beta) = -\epsilon_t$$  \hspace{1cm} (11)

subject to

$$g_1 = H_{l-1} - H_l = 0,$$  \hspace{1cm} $g_2 = H_l - H_d = 0,$  \hspace{1cm} $g_3 = H_{l+1} - H_l = 0,$  \hspace{1cm} $g_4 = \zeta(x_{target}, t_{target}) - \zeta_d = 0,$  \hspace{1cm} $g_5 = \max \left\{ |x_b(t)| \right\} - x_{max} \leq 0,$  \hspace{1cm} $g_6 = \max \left\{ |x_b(t)| \right\} - a_{max} \leq 0,$  \hspace{1cm} $g_7 = \max \left\{ |x_b(t)| \right\} - \delta_{max} \leq 0,$  \hspace{1cm} $g_{7+n+j} = -\pi - \beta_j \leq 0,$  \hspace{1cm} $j = 1, \ldots, n$  \hspace{1cm} $g_{7+n+j} = -\pi + \beta_j \leq 0,$  \hspace{1cm} $j = 1, \ldots, n$

where $f(\beta)$ is the objective function to be minimized. The general aim in constrained optimization is to transform the problem into an easier subproblem that can be solved, and is used as the basis of an iterative process. A Sequential Quadratic Programming (SQP) method is used which allows to closely imitate Newton’s method.
for constrained optimization just as is done for unconstrained optimization (Fletcher, 1980; Bazaraa et al., 1993). At each major iteration an approximation is made of the Hessian matrix of the Lagrangian function \( L(\beta, \lambda) \):

\[
L(\beta, \lambda) = f(\beta) + \sum_{j=1}^{7+2n} \lambda_j \cdot g_j(\beta)
\]

(12)

using a quasi-Newton updating method. The Lagrange multipliers \( \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_{7+2n})^T \) are necessary to balance the deviations in magnitude of the objective function and constraint gradients. The Hessian matrix is then used to generate a Quadratic Programming (QP) subproblem

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{2} \delta^T H_k \delta + \nabla f(\beta_k)^T \delta \\
\text{subject to} & \quad \nabla g_j(\beta_k)^T \delta + g_j(\beta_k) = 0, \quad j = 1, \ldots, 4 \\
& \quad \nabla g_j(\beta_k)^T \delta + g_j(\beta_k) \leq 0, \quad j = 5, \ldots, 7+2n.
\end{align*}
\]

(13)

The solution is used to form a new iterate \( \beta_{k+1} = \beta_k + \alpha_k \delta_k \). The step length parameter \( \alpha_k \) is determined by a line search procedure.

For evaluating the objective function and constraints, the complex Fourier transform is generated from the amplitude and phase spectrum. Applying the IFFT algorithm yields the associated time-dependent wave train at target location. Zero-upcrossing wave and crest heights as well as the crest front steepness of the design wave are calculated. The motion of the wave board \( x_b(t) \) is determined by transforming the wave train at \( x = \text{target} \) in terms of the complex Fourier transform \( F_{\text{target}}(\omega) \) to the location of the wave generator at \( x = 0 \) and applying the complex hydrodynamic transfer function \( F_{\text{hydro}}(\omega) \) which relates wave board motion to surface elevation close to the wave generator:

\[
x_b(t) = \text{IFFT} \left[ F_{\text{target}}(\omega) \cdot F_{\text{trans}}(\omega) \cdot F_{\text{hydro}}(\omega) \right]
\]

(14)

with \( F_{\text{trans}}(\omega_j) = \exp(i \kappa_j x_{\text{target}}) \). The maximum stroke of the wave board is set to \( x_{\text{max}} = 2m \), maximum velocity to \( u_{\text{max}} = 1.3 \text{ m/s} \) and maximum acceleration to \( a_{\text{max}} = 1.7 \text{ m/s}^2 \). The optimization terminates if the magnitude of the directional derivative in search direction is less than \( 10^{-3} \) and the constraint violation is less than \( 10^{-3} \).

### NONLINEAR SIMULATION

The two dimensional fully nonlinear free surface flow problem is analysed in time domain using potential flow theory: the fluid is inviscid and incompressible, and the flow is irrotational. The atmospheric pressure above the free surface is constant and surface tension is neglected. Hence, the flow field can be described by a velocity potential \( \phi \) which satisfies the Laplace equation

\[
\nabla^2 \phi = 0
\]

(15)

for Neumann and Dirichlet boundary conditions. \( \nabla \) is the gradient operator vector. On rigid bodies, i.e. fixed walls or oscillating wave boards, the Neumann condition is \( \partial \phi / \partial n \) denoting the particle velocity at the body surface in the normal direction \( n \), pointing out of the fluid domain. At the free surface \( (z = \zeta) \) the dynamic boundary condition follows from Bernoulli equation

\[
\frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + g \zeta = 0
\]

(16)

where \( t \) denotes the time. The kinematic free surface boundary condition is

\[
\frac{\partial \phi}{\partial z} = \frac{\partial \zeta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \zeta}{\partial x}
\]

(17)

Written in Lagrangian form, the free surface boundary conditions result in

\[
\frac{Dx}{Dt} = \frac{\partial \phi}{\partial x}, \quad \frac{Dz}{Dt} = \frac{\partial \phi}{\partial z}
\]

(18)

\[
\frac{D\phi}{Dt} = \frac{1}{2} \nabla \phi \cdot \nabla \phi - g \zeta.
\]

(19)

At each time step the velocity potential \( \phi \) is calculated in the entire fluid domain using the finite element method. From this solution the velocities at the free surface are determined by second-order differences. The normal surface elevation \( \zeta \) as well as the associated velocity potential \( \phi \) at the free surface (Dirichlet boundary condition) are calculated using Eqs. (18) and (19). To develop the solution in time domain the fourth-order Runge-Kutta formula is applied. At each time step a new boundary-fitted mesh is created. The procedure is repeated until the desired time step is reached, or the transient wave train becomes unstable and breaks.

The finite element method used in the numerical simulation was developed by Wu and Eatoek Taylor (1994, 1995). The fluid domain is divided into triangular finite elements with a total of \( m \) nodes. The velocity potential \( \phi \) is described in terms of nodal values of the potential \( \phi_j \) and linear shape functions \( N_j(x,z) \):

\[
\phi(x,z) = \sum_{j=1}^{m} \phi_j N_j(x,z)
\]

(20)

According to the Galerkin method the weighing functions are chosen to be the shape functions themselves making the residual orthogonal to the space of the shape functions. Application of the Gauss theorem gives

\[
\int_{\Gamma} N_i \frac{\partial \phi}{\partial n} d\Gamma - \int_{\Omega} \nabla N_i \sum_{j=1}^{m} \phi_j \nabla N_j \, d\Omega = 0
\]

(21)

where \( \Gamma \) is the boundary of the fluid domain \( \Omega \). Substitution of the boundary conditions into this equation leads to

\[
\begin{align*}
\int_{\Omega} \nabla N_i \sum_{j=1}^{m} \phi_j \nabla N_j \, d\Omega - \int_{\Gamma} N_i \frac{\partial \phi}{\partial n} d\Gamma \\
= \int_{\Omega} \nabla N_i \sum_{j=1}^{m} \phi_j \nabla N_j \, d\Omega - \int_{\Gamma} N_i \frac{\partial \phi}{\partial n} d\Gamma
\end{align*}
\]

(22)

where \( \Gamma_s \) is the free surface boundary and \( \Gamma_n \) are rigid boundaries on which the normal derivative of the potential is specified. This equation can be written in matrix form

\[
[A] \{ \phi \} = \{ B \}
\]

(23)

with coefficients

\[
A(i,j) = \int_{\Omega} \nabla N_i \nabla N_j \, d\Omega \quad (i,j \notin \Gamma_s)
\]

(24)

\[
B(i) = \int_{\Gamma} N_i \frac{\partial \phi}{\partial n} d\Gamma - \int_{\Omega} \nabla N_i \sum_{j=1}^{m} \phi_j \nabla N_j \, d\Omega \quad (i \notin \Gamma_s, j \in \Gamma_s)
\]

(25)
Since the derivatives of the linear shape functions with respect to \( x \) and \( z \) are constant, the matrix \( A \) is conveniently calculated from the areas of the triangles. The bandwidth of the symmetric, positive definite matrix \( A \) is reduced to a minimum by appropriate numbering of the nodes. Thus only the upper band has to be stored which reduces the required memory significantly. The calculation of the first term in Eq. (25) is simplified by the fact that the normal velocity along the piston-type wave generator is constant in \( z \) direction and becomes zero on the fixed side walls of the numerical wave tank. The Neumann boundary condition at the wave generator is given in terms of the first time-derivative of the wave board motion \( \dot{x}_i(t) \) which has been linearly smoothed down to zero within the first 10 s. Eq. (23) is solved by Cholesky decomposition.

The mesh generation is kept simple to reduce the computational burden and follows roughly an approach by Wu and Eatock Taylor (1994). The entire domain of the numerical wave tank is divided in \( x \) direction by \( NX + 1 \) vertical lines whose positions \( x_i \) depend on the nodes at the free surface. The locations of these nodes change with time and are calculated by Eq. (18). At the beginning of the simulation the free surface nodes are uniformly distributed in \( x \) direction. The \( NZ + 1 \) nodes in \( z \) direction are exponentially distributed

\[
z_{i,j} = -\left(d + \zeta_i\right) \frac{1 - \exp\left(-\alpha(d + \zeta_i)(NZ + 1 - j)/NZ\right)}{1 - \exp\left(-\alpha(d + \zeta_i)\right)} + \zeta_j
\]

where \( d \) represents the water depth and \( \zeta_i \) the surface elevation at node \( i \). The parameter \( \alpha \) determines the narrowness of the mesh near the free surface and is set to 2.0. The total number of nodes is \((NX + 1) \cdot (NZ + 1)\) and the number of triangular elements \(2 \cdot NX \cdot NZ\).

Clauss and Steinhagen (1999) used this numerical procedure for simulating nonlinear transient waves and validated the computations by laboratory data confirming the high accuracy of this approach.

RESULTS

In this investigation, the selected design variance spectrum is the finite depth TMA spectrum according to Eq. (7). The water depth is set to \( d = 5.5 \) m, the significant wave height to \( H_s = 0.7 \) m and the peak period to \( T_p = 4.43 \) s.

A high transient design wave within a tailored group of three successive waves in random sea is optimized according to the problem definition given in Eq. (11). The target zero-upcrossing wave height of the design wave is \( H_d = 2H_s \) with a maximum crest height \( C_d(x_{target}, t_{target}) = 0.6H_d = 1.2H_s \). Target location is at a distance of \( x_{target} = 100 \) m from the wave generator, and target time is \( t_{target} = 80 \) s. The heights of the leading and the trailing waves adjoining the design wave are set to be \( H_l = H_t = H_s \). The surface elevation is described by \( N = 512 \) data points with time step of \( dt = 0.2 \) s resulting in a time window of 102.2 s. The design variance spectrum remains unchanged and 70 components in the frequency range of \( \omega/\omega_p = 0.5 \) to \( \omega/\omega_p = 3.5 \) are considered.

As illustrated in Fig. 1, the optimization process finds local minima, i.e. a number of different wave trains, which depend on the initial phase values. Hence the random character of the optimized sea state is not completely lost.

One of the above wave regimes is analysed in more detail. Fig. 2 shows the minimization of the objective function \( f(\beta) \) which correlates with the maximization of wave front steepness in time domain \( \varepsilon_t \). The optimization terminates successfully at \( \varepsilon_t = 0.14 \) after 287 function evaluations. The magnitude of the directional derivative in search direction is \( 3.4 \cdot 10^{-4} \) and the maximum constraint violation is \( 1.2 \cdot 10^{-3} \).

Fig. 3 shows that all target features regarding global and local wave characteristics are met. The wave train as well as the height, crest and period structure is presented in the figure.
Linear wave theory is not appropriate for describing extreme waves which might become unstable and break before building up. However, the linear description of the wave evolution allows the determination of time-dependent boundary conditions required in the fully nonlinear numerical simulation.

The numerical wave tank used for nonlinear simulation is 500 m long and \( d = 5.5 \) m deep, with the finite element mesh constructed from \( NX + 1 = 1001 \) nodes in \( x \) direction and \( NZ + 1 = 14 \) nodes in \( z \) direction. For determining the Fourier spectrum of the nonlinear wave train, it is necessary to exclude all effects related to the start-up of wave generation. Thus the control signal is used twice resulting in a simulation time of 204.6 s with a linking time step of \( dt = 0.2 \) s.

Fig. 4 illustrates the associated nonlinear simulation with the finite element method and compares the results with the preceding linear analysis. Good agreement of the linear and nonlinear wave evolution is observed close to the wave generator at \( x = 5 \) m. At target location of \( x = 100 \) m the deviations have increased. Only \( 5 \) m further down, at \( x = 105 \) m, the nonlinear design wave is even more pronounced and clearly higher than predicted by linear theory. Linear and nonlinear Fourier spectra are in acceptable agreement.

The wave board motion and the associated nonlinear wave evolution \( \zeta (x, t) \) are presented in Fig. 5. The surface elevation as well as the wave height reach their maximum at about \( t = 81 \) s.

CONCLUSIONS

A tailored group of three successive waves embedded in random sea is optimized in time domain. The procedure is based on linear wave theory and optimizes an initially random phase spectrum for the given TMA spectrum. The desired wave characteristics like crest steepness, wave height and crest structure are defined by an appropriate objective function and constraints which take the maximum stroke, velocity and acceleration of the wave board into account. The optimization problem is solved by a Sequential Quadratic Programming method. The solution depends on initial phase values resulting in different wave trains for the same problem definition.

The linear wave evolution of the tailored wave group is compared with the fully nonlinear simulation which is based on potential flow theory and the finite element method. It is found that the optimization produces an acceptable overall picture whereas details of the flow are not correctly described. The nonlinear calculation provides complete data regarding velocity, acceleration and pressure distribution as well as energy flux and particle tracks etc.

Linear wave theory simplifies the calculation significantly and causes the optimization to converge rapidly. It may generate breaking waves which cannot be simulated numerically by using the finite element method based on potential flow theory. However, the combination of optimizing the linear wave train and simulating the nonlinear wave evolution is a powerful tool for generating and investigating transient design waves in random sea. Wave groups can be tailored according to specific needs of model tests, e.g. critical transient wave groups can be generated to investigate capsizing processes. In this context, further work is necessary to define dangerous wave regimes for different types of offshore structures.

The design wave train derived from linear wave theory may serve as an excellent initial guess for directly optimizing the fully nonlinear wave evolution.

REFERENCES


FIG. 3: Optimized transient design wave in tailored wave group ($H_{max} = 2 H_s, \zeta_{max} = 0.6 H_{max}$).
FIG. 4: Linear and nonlinear wave evolution of transient design wave in tailored wave group.
FIG. 5: Motion of the wave board and characteristics of the nonlinear wave evolution.